math problem solving in action

Getting Students to Love Word Problems, grades 3–5

An **Eye On Education** Book

Dr. Nicki Newton

Math Problem Solving in Action

In this new book from popular math consultant and bestselling author Dr. Nicki Newton, you'll learn how to help students become more effective and confident problem solvers. Problem solving is a necessary skill for the 21st century but can be overwhelming for both teachers and students. Dr. Newton shows how to make word problems more engaging and relatable, how to scaffold them and help students with math language, how to implement collaborative groups for problem solving, how to assess student progress, and much more.

Topics include:

- Incorporating problem solving throughout the math block, connecting problems to students' real lives, and teaching students to persevere;
- Unpacking word problems across the curriculum and making them more comprehensible to students;
- Scaffolding word problems so that students can organize all the pieces in doable ways;
- Helping students navigate the complex language in a word problem;
- Showing students how to reason about, model, and discuss word problems;
- Using fun mini-lessons to engage students in the premise of a word problem;
- Implementing collaborative structures, such as math literature circles, to engage students in problem solving;
- Getting the whole school involved in a problem-solving challenge to promote schoolwide effort and engagement; and
- Incorporating assessment to see where students are and help them get to the next level.

Each chapter offers examples, charts, and tools that you can use immediately. The book also features an action plan so that you can confidently move forward and implement the book's ideas in your own classroom. Free accompanying resources are provided on the author's website, www.drnickinewton.com.

Nicki Newton has been an educator for 28 years, working both nationally and internationally, with students of all ages. She has worked on developing Math Workshop and Guided Math Institutes around the country. She is also an avid blogger (www.guidedmath.wordpress.com), tweeter (@drnickimath), and Pinterest pinner (www.pinterest.com/drnicki7).

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Math Problem Solving in Action

Getting Students to Love Word Problems, Grades 3–5

Dr. Nicki Newton

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Dedication

I dedicate this book to my mom, pops, bigmom & daddy

Contents

eResources

Additional resources to accompany this book can be found on the author's website at www.drnickinewton.com

Meet the Author

Dr. Nicki Newton has been an educator for 28 years, working both nationally and internationally with students of all ages. Having spent the first part of her career as a literacy and social studies specialist, she built on those frameworks to inform her math work. She believes that math is intricately intertwined with reading, writing, listening and speaking. She has worked on developing Math Workshop and Guided Math Institutes around the country. Most recently, she has been helping districts and schools nationwide to integrate their State Standards for Mathematics and think deeply about how to teach these standards within a Math Workshop Model. Dr. Nicki works with teachers, coaches and administrators to make math come alive by considering the powerful impact from building a community of mathematicians that make meaning of real math together. When students do real math, they learn it. They own it, they understand it and they can do it—every one of them. Dr. Nicki is also an avid blogger (www.guidedmath.wordpress.com) and Pinterest pinner (www.pinterest.com/drnicki7).

Foreword

Problem Identified

In 2009, the Institute for Education Sciences published a practice guide entitled, *Assisting Students Struggling with Mathematics: Response to Intervention (RtI) for Elementary and Middle Schools.* One of the recommendations from the research was to provide instruction on solving word problems based on common underlying structures. What was that? As a middle-school teacher at that time, my problem was I didn't know the common structures, much less how to teach them. Digging into the research provided more insight that students could have positive results in solving word problems given explicit problem-solving instruction (Jitendra et al. [1998]; Xin, Jitendra, and Deatline-Buchman [2005]; Darch, Carnine, and Gersten [1984]). There was plenty of information on why a change was needed, however, practical teaching strategies for implementing efficient problem solving were still illusive.

Having taught math from the college level, then high school and down to middle school, I have experienced the struggle students have with solving word problems. Working with teachers at all levels as an administrator, I have also seen the struggles teachers have with teaching students to think critically and solve problems. We know how to teach basic skills; we've been doing it for decades! However, a change is needed to tap the critical thinking skills of the next generation.

In *World Class Learners: Educating Creative and Entrepreneurial Students* (2012), Yong Zhao discusses the need for students to be resourceful, creative and independent to meet the challenges of the new workplace. He states that "in a new economy we need a new type of talent," because the jobs of the future have not even been created. As teachers, we often tell students what they should know and then help them to become proficient. Our issue is that the new skill set is not a fact skill set but a creative skill set. A paradigm shift is needed to help teachers develop the creative skills of students. It makes sense that to foster the individual talents, we need to allow students to explore those talents in meaningful ways and to do so on a daily basis, not for just a project each grading period. Mathematical problem solving is a starting place to begin equipping students with this critical thinking skill. Just as skills need to be retooled to access the information age resources, so will a retooling of educators be necessary for the next shift.

Problem Solved

In 2015, I began helping elementary teachers to implement guided math. While searching for practical resources to share with teachers, I happened upon "Dr. Nicki's Guided Math Blog." Finally, some concrete assistance to help educators! I shared Dr. Nicki's book, *Guided Math in Action: Building Each Student's Mathematical Proficiency with Small-Group Instruction* (Routledge, 2013). However, my teachers needed more, so we were able to implement a professional development model where Dr. Nicki presented components of guided math, modeled guided math in classrooms, observed teachers, provided feedback and helped teachers with lesson planning. Dr. Nicki is a walking repository of math research with practical application tools for implementation. She was able to articulate the needed components of guided math with an emphasis on problem solving to engage students.

Teachers need not only to understand why problem solving is necessary but also how to implement this critical component into teaching. Now, in this new book *Math Problem Solving in Action*, Dr. Nicki provides practical strategies that remove the mystery for educators. In Chapters 3 and 4 she outlines the problem-solving structures that students can use to put a "handle" on word problems. Each chapter provides many practical tools and strategies to help both teachers and students attack problem solving in meaningful ways. Of particular help to my teachers was creating the Math Toolkits explained in Chapter 7 and focusing on improving our mathematical language when discussing problems explained in Chapter 5. Using these resources in daily lesson planning will assist elementary teachers in equipping our students with the critical thinking and problem-solving skills necessary for our successful future.

Happy problem solving!

Dr. Rhonda Wade
Principal
Brazosport ISD, Texas

Acknowledgments

I am so excited and honored to be writing another book. I could have not done this book without the help of others. So many people have helped me to write this book. My family is always there, encouraging me along the way—Marvin, Sharon, Tia, Uncle Bill, Clinese, George and all of my nieces, nephews and great nieces and great nephews and cousins. My friends always support my efforts (Kimberly, Tracie, Tammy, Demorris, Terri, Scott and Alison). My best assistant ever—Brittany! We have several people who help us as well, including Anna, Nancy, Gabby and Debbie. I am very grateful to Rhonda Wade who wrote the foreword for the book. My editor Lauren was a tremendous guide along the way. She is attentive, smart and has the patience of an angel. I am forever thankful and grateful to all the teachers and students that I worked with throughout the journey. You all make it possible. I sincerely thank everyone named and unnamed who has helped in the process.

Introduction

Problem solving should prepare students for the future. Problem solving is a big deal around the country and, in fact, around the world. It should be fun, but oftentimes it is greeted with dismay. Problem solving is about thinking, reasoning, exploring, hypothesizing and wondering. It takes dedication and perseverance. It takes "being up for the challenge." It's delicate. There are many aspects. One aspect is word problems. I am writing this book about how to get all kids to love word problems because I truly believe it is possible. But, I don't think we will ever achieve that goal if we keep doing what we've been doing—especially considering who we are working with these days. Our students are called Generation Z. Generation Z (also known as iGen, Post-Millennials and Centennials) (https://en.wikipedia.org/wiki/Generation_Z) is the first generation to be born into a world where the Internet has always existed.

> They are the first truly mobile-first generation, so they place a big emphasis on **personalization and relevance**. . . . Whereas Millennials use three screens on average, Gen Zers **use five:** a smartphone, TV, laptop, desktop and iPod/iPad . . . The average Gen Zer has the **attention span of about 8 seconds** . . .
>
> (http://www.cmo.com/articles/2015/6/11/15-mind-blowing-stats-about-generation-z.html)

Furthermore, Gen Zers are "extremely social and group oriented . . . prefer game-like learning situations . . . they place a value on the speed of their work, not accuracy" (http://thrivist.com/21-facts-about-generation-z-that-you-need-to-know/).

They are able to work with people from around the world from the chairs in their home. They will need to solve problems daily. Most of what they will do, we have yet to imagine. But what we know for sure is that they will need to know how to think out loud, explain and justify their reasoning to others and represent their thinking. So, we have to take problem solving seriously and do it often. Thomas Frey, one of the top futurist researchers in the world, noted that "the top three skills needed for the future [are] adaptability, flexibility and resourcefulness" (http://www.futuristspeaker.com/2014/11/101-endangered-jobs-by-2030/).

Given what we know, what are we doing in schools? Why do we teach students word problems? What are we trying to accomplish? Do we want students to actually learn something or are we engaging in raging acts of futility? We should be considering our students within their lived realities and teaching them to thrive in a future we can't name. Do we tap into the need for personalization and relevance? How often do we integrate technology into the teaching of problem solving? Do we allow them many opportunities to work in groups and play games around problem solving? Do we set up learning opportunities that require them to be adaptable, flexible and resourceful?

So, I have written this book to talk about ways to think about teaching word problems. There are 12 chapters.

Chapter 1: Real Stories, Deep Understanding

Chapter 1 discusses what problem solving should look like throughout the math block. It looks at making connections between students' real lives and real problems. It also discusses teaching students to persevere. We need to find ways to get students to think and persevere.

Chapter 2: The Basic Framework

Chapter 2 takes an in-depth look at schema-based problem solving and one of the landmark professional development ideas for teachers around this issue—the CGI problem types. This is important because every state basically uses this framework for its problem-solving standards.

Chapter 3: Problem Types Across the Curriculum

Chapter 3 looks at the various word problems across math domains and ways to unpack them so that they are comprehensible to students. It explores different types of schemas for thinking about and solving these problems.

Chapter 4: Structures to Scaffold Success

Chapter 4 looks at ways to scaffold word problems so that students can approach them in a calm way. Word problems in the upper elementary grades can be overwhelming. Students are juggling not only the words but also the math. There are many moving pieces and scaffolds help students to organize those pieces in doable ways.

Chapter 5: The Language of Word Problems

Chapter 5 looks at the problem with word problems, literally. A big challenge with word problems are the words! Language trips up students, often. Not only the English language learners and the students with learning disabilities but also the native English speakers struggle with the words and phrases in word problems. There are ways to help students navigate the language of word problems.

Chapter 6: Reasoning About Problems

Chapter 6 looks at the various aspects of reasoning about word problems. Students should not only be solving but also posing word problems. There are many ways to get them to do this individually, with partners and in groups.

Chapter 7: Modeling Thinking

Chapter 7 looks at ways to model and discuss word problems. Students should be using toolkits in their work with word problems. There are a variety of tools by grade level and topic. This chapter outlines some of those tools and shows how to use them and also how to scaffold student discussions around using them.

Chapter 8: Mini-Lessons: Springboards into Great Word Problem Premises

Chapter 8 explores teaching in ways that students love. This chapter is about doing some fun stuff. This chapter is about remembering that we teach children who love animals, songs, books and stories.

Chapter 9: Math Literature Problem-Solving Circles and Other Collaborative Activities

Chapter 9 is about using a variety of collaborative structures to engage students in problem solving. These structures include math literature circles, collaborative writing and storytelling as well as reworking word problems.

Chapter 10: Schoolwide Efforts

Chapter 10 discusses the 100 Word Problem Challenge, which is a contest where students engage in scaffolded word problem solving throughout the school year. This activity gets everybody excited and involved with word problems across the school throughout the year.

Chapter 11: Assessment

Chapter 11 is about assessment. Assessment is essential. We need to do it more often and in many more ways. I'm not talking about testing here, no . . . I'm talking about seeing where students are and helping them get to the next level based on that information. Students should be part of the process as well.

Chapter 12: Action Plan

In Chapter 12, we focus on the next steps. It's important to write an action plan. It's important to outline tasks and set dates to move the work forward. Change requires action.

The Stuff Life is Made of

Word problems can be great! Word problems are the stuff life is made of. If we can make connections for children between their daily lives and the problems we pose and solve in school, we will have much more success. This book is about giving students a repertoire of tools, models and strategies to help them think about, understand and solve word problems. We want to scaffold reasoning opportunities from the concrete (using objects) to the pictorial (pictures and drawings) and, finally, to the abstract (writing equations).

—Dr. Nicki Newton

1

Real Stories, Deep Understanding

We Don't Grow Cornfields in the South Bronx

Researchers have found that story problems "are notoriously difficult to solve" (Cummins, Kintsh, Reusser & Weimer, 1988). Geary (1994) found that "children make more errors when solving word problems than when solving comparable number problems" (p. 96). Koedinger and Nathan (2004) note that Gary Larson's cartoon captioned "Hell's Library" has bookshelves full of titles like *Story Problems*, *More Story Problems* and *Story Problems Galore* and is a powerful commentary on how we feel as a society about story problems.

Students hate problem solving, but it should be presented as a challenge. Students should look forward to working with word problems because it's the stuff they do every day. We simply have to *mathematize* the stuff they do every day. Students should own the problems they solve and pose. They should tell problems about their families, their friends, their daily activities, their school and their lives. If the problems made sense to the students, then students could make sense of the problems.

Often I hear people talk and write about real-life problems, and when I look at some of those problems, I think to myself, "Whose life is that?" When I am teaching in the South Bronx, a problem about rows of corn in a cornfield isn't exactly a *go-to example*. However, if I talk about getting some candy from the corner bodega (small grocery store), then everybody is with me. Now, this is not to say that they won't be able to solve the problem about the rows of corn in the Nebraska cornfield, but it is to say that the starting point isn't Nebraska.

Real-Life Examples about Real-Life Problems

A few years ago, I was in the South Bronx working with a brand-new fifth-grade teacher. I walked into his classroom and he looked visibly relieved to see me coming through the door. He said, "Hey, Dr. Nicki. We are working on dividing decimals." He pointed to a problem about knitting on the board. He then looked at me like "Help." I smiled and said, "Okay, I'm going to start with a different problem."

I sat in the wooden chair, faced the students and asked them how many stopped at the corner bodega that morning (which is a common routine among the schoolchildren). I asked if they had $3.00, how many egg sandwiches could they buy. Everybody began calculating and they figured it out. Then I posed a different problem. I asked what if I had 75 cents, how many mini bags of potato chips could I buy? The children knew instantly how many bags of chips I could get, because they do that every day.

These are real-life problems, problems that the students live in their daily lives. That's what students need—real-life problems. I gave them several problems about the bodega where they had to divide decimals, but I didn't tell them what they were doing specifically, until we were deep into it. Then I said, "Hey, what we are doing is dividing decimals. There is a way to do this with numbers. Mr. Amancio[1] is going to show you."

We ended the period with an exit slip that asked the students how they felt about dividing decimals. Most everybody said it was easy. You know why. Because it was connected to their everyday lives. Mr. Amancio eventually got to that text-book problem, but only after the students actually understood the math.

This experience resonates with the National Research Council's (NRC, 2001, p. 130) research that shows students can engage successfully in reasoning about problems when three conditions are met:

1. Students have a knowledge base.
2. Students understand the problem and are motivated by the content.
3. The context is familiar and comfortable.

Problem Solving Throughout the Math Class

Problem solving should be done every day as a whole class routine (see Figure 1.1). It should also be done in small guided math groups and math workstations. When done in whole groups, the emphasis is on developing the habits of mind and ways of doing that good problem solvers need. The focus here is not on quickly solving a problem but on going through the process of problem solving. It is a practice that is developed over time. Great problem solving is interwoven throughout the math block (see Figure 1.1).

Perseverance

One of the most important things that students need to know about problem solving is that they have to *persevere* with the problems. They need to get the

Figure 1.1 Problem Solving in Math Block

Element	When	How? What does it look like?
Problem of the Day (word problems)	Every day some work on the problem (this is longer, around 10 minutes)	Every day students take out their math problem-solving notebooks and work on their problems.
Energizers and Routines	Every day (these are short)	Reasoning Activities What doesn't belong? True or False?
Guided Math Groups	A few times a week	Differentiated based on student needs
Math Workstations	Daily (students should go to the problem-solving station every day or almost every day)	Differentiated based on student needs. Activities vary—solving problems, writing problems, sorting problems, matching problems and equations
Homework	A weekly problem-solving packet	2–3 rich tasks around the current unit of study with maybe one review problem

Adapted from Gojak (2011, p. 29)

idea that they have to stick with it and can't give up. There should be mini-lessons around perseverance. There are so many ways to teach and talk about this now with videos, picture books, songs and posters (see https://www.pinterest.com/drnicki7/growth-mindsetperseverance/). There are several great videos. The Michael Jordan/Nike commercial where he talks about being a winner because he has practiced and failed so many times is phenomenal. There are different versions of this commercial, but a really powerful one shows him messing up on the court with the following voice-over:

> I've missed more than 9000 shots in my career. I've lost almost 300 games. Twenty-six times, I've been trusted to take the game winning shot and missed. I've failed over and over and over again in my life. And that is why I succeed!
>
> (http://www.brainyquote.com/quotes/authors/m/michael_jordan.html)

What a powerful video! The discussion that pursues is the linchpin to the year. Students have to understand that problem solving is about sticking with it. There are some powerful posters like the powerful Japanese proverb: "Fall seven times, stand up eight."

One third-grade teacher said that she had to persevere to teach her students what the poster meant. Being children, at first they took the saying literally. They said that it meant if you fall down seven times, just keep standing up. She eventually got them to see the metaphor! There is a video entitled *Brain Jump with Ned the Neuron* (https://www.pinterest.com/pin/142004194478586625) where the brain is talking to the students and teaching them that when they struggle to understand, they are actually exercising the brain and helping it to grow.

Whole Group Lessons: Rethinking Problem of the Day

A problem of the day is often an exercise in answer-getting. Students are given a problem, a bit of time to work through it and then the answer is discussed. I propose we do something radically different. That we don't rush to the answer. That we do what Phil Daro says, "Delay answer-getting" (http://serpmedia.org/daro-talks/index.html). Sometimes, even give the students the answer so that part is done. Then, focus on the process of problem solving. Rather than a *problem of the day*, consider it more of like a *problem of the week*.

Students get the problem the first day, they read it, visualize it and discuss it with partners, in small groups and together with the whole class. The initial emphasis is to get everyone to understand the problem. In order to do this, they need to *visualize and summarize* the problem. That means they need to make a picture of the problem in their heads and then talk about what that picture looks like. As part of the whole class routine, students should share out their thinking and then discuss that thinking to see if everyone agrees what the problem is about. That might be all they do on day one.

The next thing the students should do is decide what type of problem it is and what they are looking for. Students should write a set-up equation with a question mark or letter for the unknown part. It is important that students can identify what they are looking for exactly and that they write a set-up equation. Students should use the letters of the things that they are working with. For example, if the problem is about marbles, then the students use an *M* to designate the unknown.

The next thing students should do is make a written plan rather than simply picking some numbers and jumping into the problem to find an answer. The practice of thinking about the problem and really deciding what it is about is powerful. Students should write what they are going to do and then do that. After they have solved using one way, they should always be encouraged to check another. This might take two or three days.

It is important for students to be able to make connections between different representations of the problem, such as equations, verbal descriptions, tables, graphs, sketches and diagrams (NCTM, 2000; CCSS, 2010). They should always read the work of others to see if it makes sense, and if it doesn't, they should have the language to challenge each other or ask for clarification.

The next thing students should do is double-double-check their work. I have found that when teachers tell students to check their work, they tend to check the math. The math could be correct and the answer wrong. For example, $3 \times 3 = 9$, but it might have been a division problem. So, ask students to check the math and check to see if the answer makes sense.

In order to do all of this and to do it well, students should use templates (see Figures 1.3, 1.4, 1.5 and 1.6). Templates scaffold the process. Pólya (1957)

Figure 1.2 Problem Solving Template

Pólya (1957)

Understand the Problem:

Discuss the problem.

State what is known.

State what you are looking for.

Comprehend what is happening in it.

Visualize the problem (make a picture in your head).

Retell the story.

Sketch it out (quick draw for 30 seconds).

Translate it in your own words.

What's important?

What people, places and things stand out?

What numbers are given?

Devise a Plan:

Which way will you solve it?

Carry out a Plan:

Do the math.

Check the work.

Looking Back:

Think about your answer.

Does it make sense?

Figure 1.3 Problem Solving Template

NAME: ____

DATE: _________________________

SOLVING MATH WORD PROBLEMS

1. **Read the problem. Visualize it (make a picture in your head of what is happening).**
2. **Decide on the type of problem. Circle the type:**
   - **Equal groups**
   - **Arrays**
   - **Compare**
3. **Write a set-up equation for your problem.**

---

4. **Model your thinking (use tables, diagrams, number lines, pictures).**
5. **Use your set-up equation to solve the problem.**
6. **Circle your answer. (Be sure to write the units.)**

---

7. **On the lines below, explain how you found your answer. Talk about the strategies you used to solve the problem. Use your math words.**

____

____

____

____

____

____

____

8. **Check your work by solving the problem using a different strategy. Does it make sense?**

Figure 1.4 Problem Solving Template

Name: ____

Date: ________________________________

Problem:	
Visualize and summarize the problem.	**Write a number sentence that shows what we are looking for.**
	Equation: How many steps is the problem?
Model your thinking (open number line, sketch, diagram, etc.). Write an equation that matches your model.	**Explain your thinking. Talk about your plan, your model and your strategies.**

Figure 1.5 Problem Solving Template

NAME: ____

DATE: _________________________

SOLVING MATH WORD PROBLEMS

<table>
<tr><td>

1. Read the problem. Visualize it (make a picture in your head of what is happening).

2. Decide on the type of problem. Circle the type:

- **Addition**
- **Subtraction**
- **Part-Part Whole**
- **Compare**

3. Write a set-up equation for your problem.

</td></tr>
<tr><td>

4. Model your thinking (use tables, diagrams, number lines, pictures).
5. Use your set-up equation to solve the problem.
6. Circle your answer. (Be sure to write the units.)

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7. On the lines below, explain how you found your answer. Talk about the strategies you used to solve the problem. Use your math words.

____
____
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____
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8. Check your work by solving the problem using a different strategy. Does it make sense?

</td></tr>
</table>

Figure 1.6 Problem Solving Template

Name: ____
Date: ________________________________

Problem: Is it 1 step or 2 steps? Which operations does it require (+, −, ×, ÷)?	
Visualize and summarize the problem.	**Write a number sentence that shows what we are looking for.**
	Equation: How many steps is the problem?
Model your thinking (open number line, sketch, diagram, etc.). Write an equation that matches your model.	**Explain your thinking. Talk about your plan, your model and your strategies.**

first laid out the process for us with his problem-solving questions involving the four phases (see Figure 1.2). These templates build on Pólya's original phases (see Figure 1.2). They help to scaffold the thinking and with use over time the students begin to internalize the process. Eventually, the templates are phased out. Have anchor charts set up that talk about the process of problem solving. Also, have the students make their own mini anchor charts. Once the template use is phased out, still have the students sketch out the template into their problem-solving notebooks. When students are organized, they are much more likely to succeed.

Problem Solving in Guided Math Groups

After the mini-lesson for the day, students settle into the student work period. Students will go to workstations or teacher-led groups. Sometimes in those teacher-led groups, students will do problem solving (see Figure 1.7). These lessons are much more focused and intense and differentiated toward the needs of the students in the group. These lessons could be concept lessons, procedural lessons, strategy lessons or reasoning lessons.

Figure 1.7 Guided Math Problem Solving Lessons

Concept	In this lesson, students are trying to understand what the different types of problems are. Some students may be working on change unknown problems and other students may be working on part-part-whole problems. The teacher knows the students and can shape the problem to fit the levels that students are at. This is important because problems should be scaffolded. You want students to be able to unpack the problem so that they can reason about what they are looking for. They have a much better chance of finding it if they know what they are looking for.
Procedural	Procedural problems teach students how to do something. How do we add fractions? What are the rules and procedures for doing that? How do we multiply decimals by modeling it on the number line? How can we model the subtraction of decimals with decimal grids? How do we show it with numbers?
Strategic	Strategic lessons are about students using different strategies to solve problems. Students should be familiar with a variety of strategies and know when and how to use them when problem solving.
Reasoning	Reasoning problems involve students thinking about, justifying, explaining and challenging the thinking of others and defending their own thinking,

Power Questions

Good questions are the building blocks of good problem solving. We shouldn't ever give answers. We should only scaffold thinking with good questions (see Figure 1.8). When a child says they don't know, always ask them to look in their toolkit (an actual one with tools appropriate to the grade) or use a template (an actual one that is part of their toolkits). Never ask someone else to help because the minute you do this, you just taught the child who had the question that they don't have to persevere. You have in essence said, "Don't stick with it, I'll send someone in to save you." You didn't intentionally say it, but that is the message received. This is how students learn helplessness. Instead, when a child says they are stuck, say, "What could you use to help?" If you need to, suggest a starting point. But never overscaffold. Sometimes teachers will say, "Take out the fraction strips. Grab the red trapezoid for ½ and the 3 green triangles for $\frac{3}{6}$ and see how much those make together." Okay, if you do that you

Figure 1.8 Problem-Solving Questions

Visualize	If this were a commercial, what do you see?
Summarize	What is this problem about? Do you understand the problem?
Write the set-up equation	What are we looking for? What is missing? How will you write that equation?
Make a plan	What type of problem is this? How many steps is it going to take to solve this problem? What is your plan?
Solve one way	How are you going to solve this problem? What models will you use? What strategies will you use?
Check another	How are you going to check this problem? Did you use numbers, drawings, diagrams, tables or acting out to solve this problem?
Double-double-check	Did you check the math and the answer? Does the answer make sense?
Explain your thinking	Did you write down what you did?
Think about the thinking of others	Who did it the way John did? Who did it a different way? Who agrees with John? Does anybody disagree with John?
Scaffolding questions	What tool could you use to solve this problem? What template could you use to help solve this problem? What model could you use to solve this problem? What strategy could you use to solve this problem?

are *guilty of overscaffolding*. Don't overscaffold. Let your students think. Let them wrestle with the problem. Let them figure out that they can figure it out!

Key Points

- Daily Practice
- Guided Math Groups
- Math Workstations
- Process:
    - Visualize
    - Summarize
    - Make a Plan
    - Solve One Way
    - Check Another
    - Double-Double-Check
    - Explain
- Templates

Summary

Problem solving is about helping students to see how math is part of our everyday lives. The goal is to foster flexibility, competence and confidence. Since the process is involved, problems should be initially scaffolded with templates that are eventually internalized so that students can organize their thinking. Problem solving should be done throughout the workshop from routines, through guided math lessons and in math workstations. Teachers should use questions as powerful tools to scaffold student work around problem solving.

Reflection Questions

1. Do you teach real-life problems or problems that students actually live?
2. Do you do problem-solving daily with an emphasis on building the *habit of mind* rather than on *getting the answer*?
3. Do you use templates to scaffold the process?
4. What is your big takeaway from this chapter? What one thing in your teaching will you start or expand?

Note

1. pseudonym

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2

The Basic Framework

CGI—Starting with Structure

All state standards use some type of schema-based framework for story problems. A schema is a way of organizing word problems by type. There has been a great deal of research on the effectiveness of schema-based word problems in teaching and learning (Willis & Fuson, 1988; Jitendra & Hoff, 1996; Fuchs, Fuchs, Finelli, Courey & Hamlett, 2004; Griffin & Jitendra, 2009). One of the most informative professional development books to teach schema-based problem solving is *Children's Mathematics: Cognitively Guided Instruction* (Carpenter, Fennema, Franke, Levi & Empson, 2014). The research from the book frames problem solving around getting students to understand the different problems conceptually so that they can reason, use efficient strategies and have procedural fluency.

Furthermore, the research makes the case that the key word method should be avoided! Students should learn to understand the problem types and what they are actually discussing rather than "key word" tricks. The thing about key words is that they only work with simplistic problems, and so as students do more sophisticated work with word problems, the key words do not serve them well. They may actually lead them in the wrong direction, often encouraging the wrong operation. For example, consider this problem: *John has 2 apples. Kate has 3 more than he does. How many do they have altogether?* Many students just add 2 and 3 instead of unpacking the problem. Here is another example, given this problem: *Sue has 10 marbles. She has 2 times as many marbles as Lucy. How many marbles does Lucy have?* Oftentimes, students just multiply because they see the word *times*, instead of reading and understanding the problem.

Introduction to the Types of Problems

There are four general categories for addition and subtraction problems. In third through fifth grades, the students should be able to solve all of the problem types using larger whole numbers, fractions and decimals.

Adding To Problems

Adding To problems are all about adding. There are three types (see Figure 2.1). The first type of an *Adding To* problem is where the result is unknown. For example, *The stadium had 3,050 people in it. Then, 5,789 more people came. How many people are in the stadium now?* In the upper elementary grades, the problems also involve fractions and decimals. For example,

Figure 2.1 Adding To Problems

Problem Types	**Result Unknown**	**Change Unknown**	**Start Unknown**
Join/Adding to	Marco had 5 marbles. His brother gave him 5 more. How many does he have now?	Marco had 5 marbles. His brother gave him some more. Now he has 10. How many did his brother give him?	Marco had some marbles. His brother gave him 5 more. Now he has 10. How many did he have in the beginning?
Bar Diagram Modeling Problem	? ↔ [5 | 5]	10 ↔ [5 | ?]	10 ↔ [? | 5]
What are we looking for? Where is *X*?	Both addends are known. We are looking for the total amount. The result is the unknown. In other words, we know what we started with and we know the change, we are looking for the end.	The first addend is known. The result is also known. We are looking for the change. The change is unknown. In other words, we know what happened at the start and we know what happened at the end. We are looking for the change. We need to find out what happened in the middle.	The second addend is known. The result is known. We are looking for the start. The start is unknown. In other words, we know the change and we know the end but we don't know what happened at the beginning.
Algebraic Sentence	5 + 5 = ?	5 + ? = 10 10 – 5 = ?	x + 5 = 10
Strategies to Solve	Add/ Count up	Count up/ Subtract	Count up/ Subtract
Answer	He has 10 marbles now.	He gave him 5 marbles.	He had 5 marbles in the beginning.

Rubio ate ¾ of his candy bar in the morning and ⅛ in the evening. How much of his candy bar did he eat altogether? In this problem the result is unknown. Teachers tend to tell these types of problems. They are basic and straightforward. The teacher should start with concrete items, then proceed to drawing out the story, then to diagramming the story and finally to using equations to represent the story. This is the easiest type of story problem to solve.

The second kind of *adding to* problem is the "change unknown" problem. For example, *The toy store had 509 marbles. It got some more. Now the toy store has 823 marbles. How many marbles did the toy store get?* Another example, *Maria walked 2.45 of a mile in the morning. In the afternoon she walked some more. By the end of the day, she had walked 5 miles. How far did she walk in the afternoon?* In this type of problem, the students are looking for the change. They know the start and they know the end, but they don't know the *change*.

The third type of *adding to* problem is a "start unknown" problem. For example, *The toy store had some marbles. They got 345 more on Wednesday. Now they have 1,000 marbles. How many did they have in the beginning?* In this type of problem, the students are looking for the start. This is the hardest type of *adding to* problem to solve and it takes a great deal of modeling.

Taking From Problems

Taking From problems are all about subtracting. There are three types (see Figure 2.2). The first type is *Taking From* problems where the result is unknown. For example, *The bookstore had 1,997 magazines. Then they sold 1,768 more. How many do they have now?* Another example, *John had $20. He spent $3.45. How much does he have left?* In this problem, the result is unknown. Teachers often tell these types of problems. They are basic and straightforward.

The second kind of *Taking From* problem is the "change unknown" problem. For example, *The toy store had 507 toy cars. They sold some. They have 199 left. How many did they sell?* Another example, *There was ½ a pan of brownies left. The kids ate some. Now there is only ¼ a pan of brownies left. How much of the brownies did the kids eat?* In this type of problem, the students are looking for the change. They know the start and they know the end, but they don't know the *change.*

The third type of *Taking From* problem is a "start unknown" problem. For example, *Jenny had some money. She gave John $3.75. Now she has $7.30 left. How much money did she have in the beginning?* In this type of problem, the students are looking for the start. This is the hardest type of *Taking From* problem to solve and it takes a great deal of modeling.

Figure 2.2 Taking From Problems

Problem Types	**Result Unknown**	**Change Unknown**	**Start Unknown**
Separate/ Taking From	Marco had 10 marbles. He gave his brother 4. How many does he have left?	Marco had 10 marbles. He gave some away. Now he has 5 left. How many did he give away?	Marco had some marbles. He gave 2 away and now he has 5 left. How many did he have to start with?
Bar Diagram Modeling Problem	10 ⟷ [4][?]	10 ⟷ [?][5]	? ⟷ [2][5]
What are we looking for? Where is *X*?	In this story we know the beginning and what happened in the middle. The mystery is what happened at the end. The result is unknown.	In this story we know the beginning and the end. The mystery is what happened in the middle. The change is unknown.	In this story we know what happened in the middle and what happened at the end. The mystery is how did it start? The beginning is unknown.
Algebraic Sentence	10 – 4 = ?	10 – ? = 5 5 + x = 10	? – 2 = 5 2 + 5 = ?
Strategies to Solve	Subtract	Subtract until you have the result left/ Count up	Count up/ Subtract
Answer	10 – 4 = 6 He had 6 marbles left.	10 – 5 = 5 5 + 5 = 10 He gave away 5 marbles.	7 – 2 = 5 2 + 5 = 7 He had 7 marbles in the beginning.

Part-Part-Whole Problems

A *Part-Part-Whole* problem is a problem that discusses the two parts and the whole. There are three types of *Part-Part-Whole* problems (see Figure 2.3). The first is a problem where the *whole* is unknown. For example, *The toy store had 467 big marbles and 598 small marbles. How many marbles do they have altogether?* Another example, *John had $597.09 in his bank account and $28.34 in his piggy bank. How much money does he have altogether?* We know both parts, and the task is to figure out the whole.

Figure 2.3 Part-Part-Whole Problems

Problem Types	**Quantity Unknown**	**Part Unknown**	**Both Addends Unknown**
Part-Part-Whole/ Putting together/ Taking Apart	Marco has 5 red marbles and 5 blue ones. How many marbles does Marco have? 5 + 5 = x	Marco has 10 marbles. Five are red and the rest are blue. How many are blue? 10 – 5 = or 5 + x = 10	Marco has 10 marbles. Some are red and some are blue. How many could be red and how many could be blue?
Bar Diagram Modeling Problem	? 5 | 5	10 5 | ?	10 ? | ? *Often all of the combinations are represented in a table
What are we looking for? Where is *X*?	In this type of story, we are talking about a group, set or collection of something. Here we know both parts and we are looking for the total.	In this type of story, we are talking about a group, set or collection of something. Here we know the total and one of the parts. We are looking for the amount of the other part.	In this type of story, we are talking about a group, set or collection of something. Here we know the total but we are to think about all the possible ways to make the group, set or collection.
Algebraic Sentence	5 + 5 = ?	5 + ? = 10 10 – 5 = ?	x + y = 10
Strategies to Solve	Add/ Count up	Count up/ Subtract	Count up/ Subtract
Answer	5 + 5 = 10 He had ten marbles.	5 + 5 = 10 10 – 5 = ? Five were blue.	1+9 4+6 9+1 6+4 2+8 5+5 8+2 3+7 10+0 0+10 7+3 These are the possibilities.

The second kind of *Part-Part-Whole* problem is where one of the *parts* is unknown. For example, *The toy store had 1,000 marbles. There were 897 small marbles. The rest of the marbles were large. How many large marbles did they have?* Another example, *Kelly had $500. She had $257 in her piggy bank and the rest in her bank account. How much does she have in her bank account?*

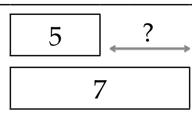
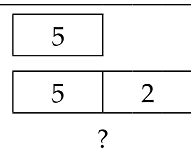
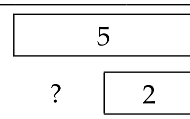
In this type of problem, we are given the whole and one of the parts. The task is to figure out the other part.

The third type of part-part-whole problem is a *Both Addends Unknown* problem. In this type of problem both addends are not known; only the total is given. For example, *Jane has 25 cents. Name all the possible coin combinations that she could have*. The task is to figure out all of the possible combinations.

Compare Problems

Compare problems are the most difficult types of stories to tell. There are three types of comparison stories (see Figure 2.4). The first type of comparison story is where two different things are being compared. For

Figure 2.4 Compare Problems

Problem Types	**Difference Unknown**	**Bigger Part Unknown**	**Smaller Part Unknown**
Compare	Marco has 5 marbles. His brother has 7. How many more marbles does his brother have than he does?	Marco has 5 marbles. His brother has 2 more than he does. How many marbles does his brother have?	Tom has 5 rocks. Marco has 2 less than Tom. How many rocks does Marco have?
Bar Diagram Modeling Problem			
What are we looking for? Where is *X*?	In this type of story, we are comparing two amounts. We are looking for the difference between the two numbers.	In this type of story, we are comparing two amounts. We are looking for the bigger part which is unknown.	In this type of story, we are comparing two amounts. We are looking for the smaller part which is unknown.
Algebraic Sentence	7 – 5 = ?	5 + 2 = ?	5 – 2 = ?
Strategies to Solve	Count up/ Count back	Count up	Subtract
Answer	His brother had 2 more marbles than he did.	His brother had 7 marbles.	Marco had 3 marbles.

example, *Susie ate ¼ of her candy bar. Joe ate ⅕ of his candy bar. Who ate more?* Another example, *Mary had $500 and Jane had $350. How much more money does Mary have than Jane?* Or, *How much less money does Jane have than Mary?* In a difference problem, when you say *less*, it is considered a more difficult version of the problem. There is another version of the compare the difference problem. For example, *Jean has 20 marbles and Mike has 10. How many more marbles does Mike need to have the same amount as Jean?*

The second type of comparison story is where the bigger part is unknown. In this type of story, we are looking for the bigger amount. For example, *Luke had $507 and Marcos had $109 more than Luke. How much does Marcos have? How much do they have altogether?* Another example, *Luke had $507. This is $109 less than Marcos. How much does Marcos have?* There are two types of this problem. When you say *less* and you are looking for more, it is considered the harder part because it is counterintuitive. The task is to find the bigger part.

The third type of comparison story is where the smaller part is unknown. In this type of story, we are looking for the smaller amount. For example, *Luke had $507 and Marcos had $109 less than Luke. How much does Marcos have? How much do they have altogether?* Another example, *Luke had $507. This is $109 more than Marcos. How much does Marcos have?* There are two types of this problem. When you say *more* and you are looking for the smaller part, it is considered the harder version because it is counterintuitive. The task is to find the smaller part.

Multiplication Problems

There are several different types of multiplication problems (see Figure 2.5). Equal group problems require students to look for the total amount of the group. Array problems require students to look for the total amount in the array. Area problems require students to solve for the total area. Rate problems are about students calculating the amount of time it takes to do something. Price problems are about calculating the cost of items. These are the base types of problems, and two-step and multistep problems are mixtures of these problems with the addition and subtraction types.

Division Problems

There are several different types of division problems (see Figure 2.6). Equal group problems require students to look for either the total amount in each group or the total amount of groups. Array problems require students to look for the amount in each row or the amount of rows. Area problems require students to solve for the sides or the total area. Rate problems are about students calculating the amount of time it takes to do

Figure 2.5 Multiplication Problems

Problem Types	**Equal Groups**	**Arrays**	**Area**	**Rate**	**Price**
Multiplication	Maribel has 4 purses. In each purse she has 5 rings. How many rings does she have altogether?	Farmer Mike had 4 rows of peach trees. He had 9 trees in each row. How many peach trees did he have altogether?	Ms. Betty had a garden that was 5 feet by 6 feet long. What was the area of her garden?	Trish runs 2 miles an hour. How many miles does she run in 4 hours?	A toy car costs $3.50. If Jamal buys 2 toy cars, how much will he spend?
Bar Diagram Modeling Problem	? | 5 | 5 | 5 | 5 |	? | 9 | 9 | 9 | 9 |	6ft. 5ft.	? | 2 | 2 | 2 | 2 |	| $3.50 | $3.50 |
What are we looking for? Where is *X*?	In this type of story, we are looking for the total amount.	In this type of story, we are looking for the total amount.	In this type of story, we are looking for the area.	In this type of problem, we are looking for the rate.	In this type of problem, we are looking for the total price.
Algebraic Sentence	$5 + 5 + 5 + 5$ $4 \times 5 = 20$	$9 + 9 + 9 + 9$ $4 \times 9 = 36$	$5 \times 6 = 30$	$2 + 2 + 2 + 2$ 4×2	$3.50 + 3.50 = 7.00$ $3.50 \times 2 = 7.00$
Strategies to Solve	Repeated Addition Multiply	Repeated Addition Multiply	Multiply	Repeated Addition Multiply	Repeated Addition Multiply
Answer	Maribel had 20 rings.	Farmer Mike had 36 peach trees.	The garden is 30 square feet.	Trish runs 8 miles in 4 hours.	Jamal spent $7.

Figure 2.6 Division Problems

Problem Types	Amount in Each Equal Group	Amount of Equal Groups	Arrays How many in each group?	Arrays How many groups?	Area	Rate	Price
Division	Grace had 20 rings that she divided up into 2 boxes. How many rings did she put in each box?	The bakery made 50 cupcakes. They packed 10 per box. How many boxes did they use?	Farmer John planted a total of 44 peach trees in 4 rows. How many trees were in each row?	Farmer John planted 44 peaches in rows with 11 peaches in each one. How many rows did he plant?	Ms. Betty had a garden that covered 30 square feet. It was 5 feet long. What was the width?	James rode his bike 5 miles in 1 hour. If he wants to ride 10 miles today, how long will it take him?	Toy cars cost $7 for a pair. How much does each car cost?
Bar Diagram Modeling Problem	20 ? ?	50 10 → ?	? ? ? ? 44	44 11 ?	? 30 square feet 5 ft.	10 5 → ?	7 ? ?
What are we looking for? Where is *X*?	In this type of story, we are looking for the amount in each group.	In this type of story, we are looking for the amount of groups.	In this type of story we are looking for the amount in each row.	In this type of story, we are looking for the amount of rows.	In this type of story, we are looking for one of the sides.	In this type of problem, we are working with rates.	In this type of problem, we are looking for the price of each item.
Algebraic Sentence	20 ÷ 2 = 10	50 ÷ 10 = 5	44 ÷ 4 = 11	44 ÷ 11 = 4	30 ÷ 5 = 6	10 ÷ 5 = 2	7 ÷ 2 = 3.50
Strategies to Solve	Divide Think multiplication	Divide Think multiplication	Divide Think multiplication	Divide Think multiplication	Divide Think multiplication	Divide Think multiplication	Divide Think multiplication
Answer	Grace had 10 rings in each box.	The bakery used 5 boxes.	Eleven trees were in each row.	He planted 4 rows.	The width was 6 feet long.	It will take him 5 hours.	Each car cost $3.50.

something. Price problems are about calculating the cost of items. These are the base types of problems, and two-step and multistep problems are mixtures of these problems with the addition and subtraction types.

Multiplicative Comparison Problems

There are three different types of multiplicative comparison problems (see Figure 2.7 and Figure 2.8). The first type is where we are looking for the bigger part. We are comparing two quantities and looking at a multiplicative difference. For example, *John has 40 marbles; his cousin David has 3 times as many as he does.* In the second type of problem, we are looking for the smaller part. For example, *David has 120 marbles, he has 3 times as many as his cousin John. How many does John have?* This is a tricky problem for students because they want to multiply instead of divide because they see the word *times* and want to solve it, without thinking about the entire problem. In the third type of problem, we are looking for the difference between the two parts. This problem tends to be the hardest to understand. It is where both quantities are given and students have to think about the relationship between the two numbers. For example, *Mike has 15 toy cars. John has 3 toy cars. How many times as many toy cars does Mike have as John?*

Figure 2.7 Additive Comparison and Multiplicative Comparison Problems

> Make sure that you emphasize the difference between additive comparison problems and multiplicative comparison problems. Illustrative Mathematics has a really good snake problem where they discuss measuring different-sized snakes that grow the same amount. They ask the students to explain if the snakes grew the same amount and expect students to be able to discuss that there are a few ways to look at it—both additively and multiplicatively.
>
> ***See Illustrative Mathematics Snake Problem 1 & 2
>
> https://www.illustrativemathematics.org/content-standards/tasks/356
>
> https://www.illustrativemathematics.org/content-standards/tasks/357

CGI and Fraction Word Problems

CGI also has a hierarchy of word problems for fractions and decimals. This is amazing, although not many people are aware of it or use it in their day-to-day curriculum approaches. Empson and Levi (2011) wrote an incredible book describing the continuum of teaching fraction and

Figure 2.8 Multiplicative Comparison Problems

Problem Types	**Bigger Part Unknown**	**Smaller Part Unknown**	**Difference Unknown (Number Times as Many)**
Compare	Marco has 5 marbles. His brother has 2 times as many as he does. How many more marbles does his brother have than he does? How many do they have altogether?	Luke has 10 marbles. He has 2 times as many as Marco. How many does Marco have?	Marco has 5 marbles. Luke has 10 marbles. How many times as many marbles does Luke have as Marco?
Bar Diagram Modeling Problem	5 5 | 5 ?	5 | 5 5	5 5 | 5
What are we looking for? Where is *X*?	In this type of story, we are comparing two amounts. We know the smaller amount and we have to find the larger amount.	In this type of story, we are comparing two amounts. We are looking for the smaller part which is unknown.	In this type of story, we are comparing two amounts. We know both parts but we are looking for *how many times as many* the larger part is than the smaller part.
Algebraic Sentence	5 × 2 = 10 10 + 5 = 15	10 ÷ 2 = 5	5 × ? = 10 or 10 ÷ 5 = 2
Strategies to Solve	Multiply Addition	Divide	Multiply or divide
Answer	His brother had 10 marbles. Altogether they had 15 marbles.	Marco had 5 marbles.	Luke had 2 times as many marbles as Marco.

decimal word problems called *Extending Children's Mathematics: Fractions & Decimals: Innovations in Cognitively Guided Instruction.*

Many state standards somewhat follow the trajectory; however, teachers tend to start with whatever standards are in their grade. It is of the utmost importance that teachers figure out where their students are and then start from there. It is crazy-making to just start where the book does. Often, when we start where the book does, instead of where the students are, we get big problems. We say, "These students don't understand fractions." The intervention should start at the level the student is at, and that requires that we assess students and then provide intentional interventions. The figure below (Figure 2.9) shows a continuum of a fraction hierarchy that is followed in most state standards. There are a few differences in the order in some states, but this is the general hierarchy. What is important about this is that it provides a framework for testing prior knowledge before jumping into a fraction unit.

Figure 2.9 Fraction CGI Problems

Level 1: Start with situations of 2 or 4 children, as children's earliest partitions are based on halving	**Level 2:** Move to situations of three sharers	**Level 3:** Move into equivalent fraction problems	**Level 4:** Move to higher levels of sharers	**Level 5:** Comparisons of equal-sharing situations
Level 6: Adding fractions with like denominators	**Level 7:** Adding mixed numbers with like denominators	**Level 8:** Subtracting fractions with like denominators	**Level 9:** Subtracting mixed numbers with like denominators	**Level 10:** Multiplying a fraction by a whole number
Level 11: Adding fractions with unlike denominators	**Level 12:** Adding mixed numbers with unlike denominators	**Level 13:** Subtracting fractions with unlike denominators	**Level 14:** Subtracting mixed numbers with unlike denominators	**Level 15:** Multiplying 2 fractions
Level 16 Dividing a whole number by a fraction	**Level 17** Dividing a fraction by a whole number	**Level 18** Dividing 2 fractions	**Level 19** More complex multistep problems	

Two-Step Problems and More

After students have mastered solving one-step problems, they start working on two-step problems and then multistep problems (see Figures 2.10 and 2.11). There is still a level of hierarchy that is often neglected. This is highly detrimental to student learning because two-step and multistep problems are simply one-step problems with two or more parts. It is important to remember to scaffold levels of difficulty so that the cognitive load is balanced. Don't give hard problems with hard numbers to start with because then students become cognitively overloaded.

Give hard problem types with easy numbers so that students can focus on the problem. Once they know how to solve the problem, then give harder numbers. Often the problem is that students don't fully understand one of the parts. So it is crucial that students understand the one-step problems before they go on to others (see Figures 2.10 and 2.11). This must be assessed and addressed on an ongoing basis.

How Do We Teach This?

These are a lot of different problem types. The research states that teachers should teach the types explicitly to the students and that the students

Figure 2.10 Two-Step Problems

Level 1	**Level 2**	**Level 3**	**Level 4**	**Level 5**
Same Operation	Different Operations	Comparison	Mixed Levels	Mixed Levels of Harder Versions
Sue had $5.07. For her birthday, her mother gives her $14.09 and her brother gives her $10.89. How much money does she have now?	Sue had $5.07. She gives her sister $2.90. Her dad gives her $4.52 more. How much money does she have now?	Sue had $35. Her sister had $3 less than she did. How much money did they have altogether?	On the farm there were 700 animals. There were 250 sheep and some horses. Then, the farmer bought 25 more horses. How many horses are there now? How many animals are there altogether now?	On the farm there were 700 animals. There were 250 sheep and some horses. The farmer bought more horses. Now there are 900 horses. How many horses did the farmer buy? How many animals are there altogether now?

Figure 2.11 Multistep Problems

Type A	Type B	Type C	Type D	Type E
Mike had 5 marbles. Jake had 2 times as many as Mike. How many did they have altogether? If Jake got 5 more marbles, now how many times as many marbles as Mike will he have? On the farm there were 700 animals. There were 250 sheep, some horses and some cows. There were 100 more sheep than cows and the rest of the animals were horses. Then, the farmer bought 25 more horses. How many horses are there now? How many animals are there altogether now?	It cost $5 for students and $10 for adults to go to the museum. Class 305 is taking 25 students and 4 adults to the museum. If they have raised $125 will they have enough? On the farm there were 700 animals. There were 250 sheep, some horses and some cows. There were 100 more sheep than cows and the rest of the animals were horses. The farmer bought more horses. Now there are 500 horses. How many horses did the farmer buy? How many animals are there altogether now?	The bakery sold 49 cookies on Monday, 56 cookies on Tuesday, and 63 cookies on Wednesday. If the pattern continues, how many cookies will they have sold on Sunday? What is the total amount of cookies they will have sold in a 7-day week?	The store sells sugar in 1-lb, 5-lb and 10-lb bags. What is the fewest number of bags to buy to get 27 lbs of sugar?	My sister had 45 cents. She had a quarter. What other coins could she have had?

should be able to name the type of problem they are solving. The terminology isn't that important. It is the concept that teachers should emphasize and that students should know. Students need to know that they are putting stuff together or taking it apart. They should understand whether or not they are looking for the amount in each group or if they are looking for the amount of groups. They should think about whether this is a one-step, two-step or multistep problem. This just helps students to unpack a problem and make a plan for solving it. Because if students know what they are looking for, then they are much more likely to find it!

Word Problems in the 21st Century

Students can either do some Web-based activities in the word problem workstation or in the digital workstation (see Figure 2.12). There are some great sites for students to practice word problems. Math Playground has *Thinking Blocks*, where students can work through a series of models for tape diagramming problems based on the operation. This is a fantastic site: Math Playground: http://www.mathplayground.com/ThinkingBlocks/thinking_blocks_start.html.

Greg Tang has also recently built a whole world of word problem activities where students can pick the word problem by type and even

Figure 2.12 Problem-Solving Resources

Picture Books/ Stories	Paper and Virtual Tools	Internet Word Problem Sites	Online Resources
Use picture books as a launch into different problem contexts. There is a great book called *Tall Tale Math* specifically for grades 3–5 that mathematizes tall tales.	Use a variety of physical and virtual manipulatives to solve problems. For virtual manipulatives see: http://nlvm.usu.edu/ http://www.glencoe.com/sites/common_assets/mathematics/ebook_assets/vmf/VMF-Interface.html	http://www.mathplayground.com/thinkingblocks.html https://learnzillion.com/ http://gregtangmath.com/materials http://www.mathplayground.com/wp_videos.html http://www.mathplayground.com/ThinkingBlocks/thinking_blocks_start.html	https://www.illustrativemathematics.org/ http://www.insidemathematics.org/ https://sddial.k12.sd.us/esa/grants/sdcounts/

get hints about how to set up the problem with tape diagrams. If the students do one problem at a time, they can get the hints and the answers. This is how I recommend that they practice it. There is also an option to generate several word problems at a time, but these just give the word problems without all of the scaffolding. (See the Greg Tang Word Problem Generator: http://gregtangmath.com/materials.)

Key Points

- CGI provides a framework for teaching word problems (Carpenter et al., 2014).
- The emphasis should be on the problem types and structure rather than on key words.
- There are four major categories for addition and subtraction.
- There are three major categories for multiplication and division.
- There are multiple levels of fraction problem solving.
- There are levels for two-step problems.
- There are different types of multistep problems.
- There are many 21st century tools to teach word problems.

Summary

Schema-based problem solving is a framework that is used to teach word problems (Carpenter et al., 2014). Many educators did not learn word problems from this framework and need to take the time to understand them so that they can better teach them. Students should learn to think about the structure and type of problem so they can set it up accordingly. Students should also focus on how many steps the problem involves so that they can prepare for that in the planning and double-check all the parts in the end. There are many great resources to teach problem solving and to practice problems these days that are free and valuable.

Reflection Questions

1. Do you presently teach with the word problem types in mind?
2. Do you presently get students to think about the steps involved in the word problem? If so, how do you currently do it? If not, how might this help?
3. Do you use technology to teach and practice word problems?
4. What are your biggest takeaways from this chapter?

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3

Problem Types Across the Curriculum

There are many opportunities to engage in problem solving across the math curriculum (see Figure 3.1). There are the general content standards and then the specific word problem standards. In this chapter, we will look at the specific problem-solving standards across the curriculum including money, time, capacity, mass, fractions and decimals. This chapter is not meant to be exhaustive but rather to give examples of what unpacking problems across the curriculum might look like. Each topic has its own set of wiggly issues, so it is important to think about how we are framing the issues for the students. Here are possible ways to frame problems so that students are better able to approach them.

Operations and Algebraic Thinking

Multiplication and Division Problems with Picture Prompts

Using picture prompts helps students to visualize the problem and therefore scaffolds their thinking. Look at Figure 3.2 for an example. Students can actually use the pictures to work out the problem. This should be done sometimes, but it also helps to scaffold student modeling because students should be able to actually sketch out their own models eventually.

Measurement Problems

Money Problems

Money problems are difficult for students. But money is a part of students' everyday lives. Make sure that you use play money to teach these stories. Students need to actually hold and count and reason about money by using play money. They need to do two- and three-step problems involving all of the operations by actually acting them out. First, we need to teach students to identify what type of problem they are working with (see Figures 3.3, 3.4, 3.5, 3.6, 3.7 and 3.8). Next we need to teach them how to model the problem in different ways.

Figure 3.1

Grade 5					
Operations and Algebraic Thinking					
	Represent and solve word problems that involve two numerical patterns and rules. Graph these problems. Students should be able to recognize the difference between additive and multiplicative numerical patterns given in a table or graph.	Represent and solve problems that involve comparing and ordering two decimals to the thousandths and represent comparisons using the symbols < , > or =.		Represent and solve word problems that involve estimating and rounding decimals to tenths or hundredths.	Represent and solve multistep problems involving the four operations with whole numbers using equations with a letter for the unknown quantity.
Place Value					
	Represent and solve word problems with the products up to 3 digits by a 2-digit number using various models including arrays, area models or equations. Use a variety of strategies including mental math, partial products, and the commutative, associative, distributive properties and the standard algorithm.	Represent and solve multi-digit division problems with up to 4-digit dividends and 2-digit divisors using arrays, area models or equations. Use a variety of strategies *(*including the standard algorithm)*. **Teks*	Represent and solve for division decimal word problems with quotients of decimals to the hundredths, up to 4-digit dividends and 2-digit whole number divisors, using objects and pictorial models, including area models. Use a variety of strategies and algorithms including the standard algorithm.	Represent and solve decimal multiplication word problems with products to the hundredths using objects and pictorial models, including area models. Also, use a variety of strategies based on place value, properties of operations and the relationship to whole numbers. Include situations involving money.	Estimate to determine solutions to real-world problems involving the four operations.

Fractions			
	Represent and solve word problems involving addition and subtraction with unlike denominators using a variety of strategies, models and properties of operations.	Represent and solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers.	Represent and solve real world problems involving multiplication of fractions and mixed numbers, and fractions and whole numbers. **Teks*: Represent and solve multiplication of a whole number and a fraction that refers to the same whole using objects and pictorial models, including area models.
	Represent and solve word problems that involve dividing unit fractions by whole numbers and whole numbers by unit fractions using concrete, pictorial and abstract models.	Create word problems that involve the division of a whole number by a unit fraction using concrete, pictorial and abstract models.	Represent and solve word problems involving like, unlike, mixed and improper fractions.
Measurement			
	Represent and solve one-step and two-step word problems that involve converting measurement units within either the customary or metric system. Kilometers and meters Meters and centimeters Kilograms and grams Liters and milliliters	Represent and solve word problems about volume. Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real-world and mathematical problems.	**Teks*: Represent and solve problems related to perimeter and/or area.

(*Continued*)

Figure 3.1 (Continued)

Data			
	Represent and solve line plot problems involving fractions of a units (½, ¼, ⅛).	Represent and solve one-step and two-step problems using data from bar graphs or frequency tables,* dot plots or stem-and-leaf plots or scatterplots (including data sets of measurements in fractions or decimals.) **Line graphs CCSS **Teks* (only) **Nebraska (probability problems)	
Geometry			
	Represent and solve word problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation, including those generated by number patterns or found in an input-output table.		

Grade 4				
Operations and Algebraic Thinking.				
	Represent and solve multiplicative comparison word problems with drawings and equations with a symbol for the unknown number to represent the problem	Represent and solve one-step and two-step word problems with whole numbers using all the four operations, including problems with remainders. Use estimation, equations and rounding in solving the problems. Use a variety of models, including strip diagrams and equations with a letter standing for the unknown.	Represent and solve word problems about factors.	Represent and solve pattern problems. Use an input-output table.
	Represent and solve word problems involving the product of two 2-digit numbers using arrays, area models or equations, *(*including perfect squares through 15 by 15)*. **Teks*	Represent and solve word problems using strategies and algorithms *(*including the standard algorithm)* to multiply up to a 4-digit number by a 1-digit number and to multiply a 2-digit number by a 2-digit number. Strategies may include mental math, partial products, and the commutative, associative and distributive properties. **Teks*	Represent and solve word problems involving the quotient of up to a 4-digit whole number divided by a 1-digit whole number using arrays, area models, or equations. Use strategies and algorithms, *(*including the standard algorithm)*.	Represent multistep problems involving the four operations with whole numbers using strip diagrams and equations with a letter standing for the unknown quantity.

(*Continued*)

Figure 3.1 (Continued)

Place Value				
	Represent and solve multi-digit word problems that are multistep. Solve with fluency one-step and two-step problems involving multiplication and division including interpreting remainders.	Represent and solve word problems that involve rounding to the nearest 10, 100 and 1,000.	Represent and solve the product of a 1-digit number by a 2-, 3- and 4-digit number as well as the product of two 2-digit numbers using various models including arrays, area models, or equations. Use a variety of strategies including mental math, partial products, and the commutative, associative and distributive properties.	Represent and solve word problems with 1-digit divisors by 2-, 3- and 4-digit dividends using arrays, area models or equations. Use a variety of strategies.
Fractions/Decimals				
	Represent and solve word problems that involve adding 2 fractions with like denominators. Use objects, pictorial models, number lines and properties of operations.	Represent and solve word problems that involve subtracting 2 fractions with unlike denominators. Use objects, pictorial models, number lines and properties of operations.	Represent and solve word problems that involve adding and subtracting mixed numbers. Use objects, pictorial models, number lines and properties of operations.	Represent and solve word problems that involve multiplying a fraction by a whole number. Use objects, pictorial models, number lines and properties of operations.
	Represent and solve word problems where students have to reason about and compare two fractions with different numerators and different denominators and represent the comparison using the symbols < , > or =.		Represent and solve word problems where students have to work with equivalent fractions.	Represent and solve word problems involving comparing and ordering decimals using concrete and visual models to the hundredths.

Decimals				
	Solve word problems adding decimals with denominators of tenths and hundredths. Use concrete visual models, number lines and money.	Solve problems that involve comparing and ordering decimals using concrete and visual models to the hundredths.		Solve word problems that involve adding and subtracting whole numbers and decimals to the hundredth place using the standard algorithm. **Teks*
Measurement				
	Solve word problems involving the two measurement systems. Solve both customary and metric problems. Students should know km, m, cm; kg, g; lb., oz. l, ml, hr, min, sec.	Represent and solve word problems that require conversions within the same measurement system, customary or metric, from a smaller unit into a larger unit or a larger unit into a smaller unit when given other equivalent measures in a table. **In some states this is fourth grade, and in others this is fifth grade.*	Solve word problem using the four operations. Problems about *(*distances)*, length, intervals of time, liquid volumes, masses of objects, and money, including problems *(*involving simple fractions or decimals)*. Also measurement problems involving conversions from a larger unit to a smaller unit. **CCSS*	Solve area and perimeter problems where dimensions are whole numbers using models. Use formulas for a perimeter of a rectangle, including the special form for perimeter of a square and the area of a rectangle.

(*Continued*)

Figure 3.1 (Continued)

Data				
	Solve problems involving addition and subtraction of fractions by using information presented in line plots (fraction units of (½, ¼, ⅛).		Solve one-step and two-step problems using data and whole number, decimal and fraction form in a frequency table, dot plot, stem and leaf plot marked with whole numbers and fractions. **Teks standard*	
Geometry				
	Solve addition and subtraction problems to find unknown angles.			

Grade 3				
Operations and Algebraic Thinking				
	Given a division expression or equation, write a word problem.	Given a multiplication expression or equation, write a word problem.	Represent and solve one-step and two-step multiplication and division word problems within 100 involving repeated addition, equal groups, and skip counting, properties of operations and/or recall of facts. Use various models including objects, equal jumps on the number line, arrays, area models, strip diagrams and equations.	Represent and solve pattern problems in a table.

	Represent and solve two-step word problems with whole numbers using all the four operations. Include problems that involve estimating and rounding.	Represent and solve word problems involving multiplying a 2-digit number by a 1-digit number using strategies and algorithms, including the standard algorithm that may include mental math, partial products, and the commutative, associative and distributive properties. **Teks*		
Place Value				
	Represent and solve multi-digit word problems that are two-step.	Represent and solve with fluency one-step and two-step problems involving addition and subtraction within 1,000 using strategies based on place value, properties of operations and the relationship between addition and subtraction. Represent with various models, including pictorial models, number lines and equations.		Represent and solve addition and subtraction problems that involve estimation and rounding.
Fractions				
	Represent and solve word problems about fractions as parts of wholes.	Represent and solve word problems that compare two fractions having the same numerator or denominator by reasoning about their sizes. Justify the conclusions with symbols, words, objects and pictures.	Represent and solve word problems where students have to reason about equivalent fractions with 2, 3, 4, 6 and 8.	Represent and solve problems involving partitioning an object or a set of objects among two or more recipients using pictorial representations of fractions with the denominators of 2, 3, 4, 6 and 8. **Teks*

(*Continued*)

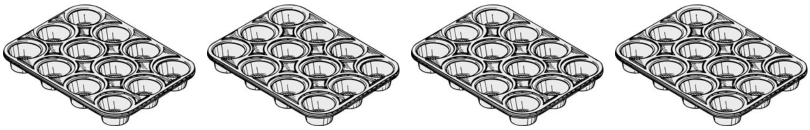
Figure 3.1 (Continued)

Measurement				
	Represent and solve word problems involving addition and subtraction of time intervals in minutes using pictorial models or tools.	Represent and solve one-step word problems about weight/ mass and volume/capacity (liquid volume) that involve addition, subtraction, multiplication or division. **Teks* ***CCSS only does mass/liquid volume*	Represent and solve area of rectangles with whole-number side lengths. Also solve area word problems about rectilinear figures.	Represent and solve word problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length and with rectangles with the same perimeter and different areas or with the same area and different perimeters.
	Represent and solve word problems involving coins and bills.			

Data			
	Represent and solve one-step and two-step word problems using information from scaled bar graphs.	Represent and solve problems involving addition and subtraction of fractions by using information presented in line plots (fraction units of ½ and ¼).	Represent and solve one-step and two-step problems using categorical data represented in a frequency table, pictograph or bar graph with scaled intervals. **Teks* includes dot plots as well.

* The word problems are drawn from the TEKs, CCSS, Nebraska and various state standards.

Figure 3.2



Melissa said she would make 4-dozen cupcakes. One-fourth of the cupcakes will be vanilla, ½ of the cupcakes will be chocolate and the rest will be strawberry.

1. How many of each will she make?

2. If ½ of the chocolate will have sprinkles, how many cupcakes will have sprinkles?

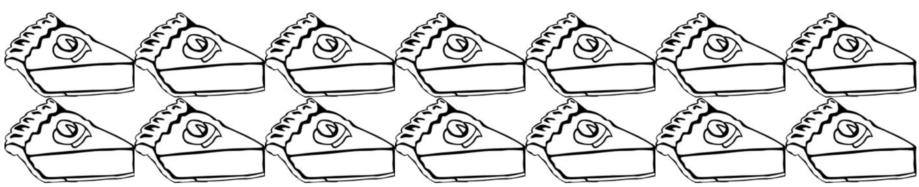
Figure 3.3 Money Word Problem Types

Result Unknown	Change Unknown	Start Unknown	Part-Part-Whole Whole Unknown	Part-Part-Whole Part Unknown
Joel had $589. He got $345 more for his birthday. How much does he have now?	Tom had $409. He saved some more. Now he has $1,000. How much did he save?	Mike had some money. He got $234 more. Now he has $700. How much did he have in the beginning?	Maribel had 35 quarters and 25 dimes. How much money did she have altogether?	Joel went to the store with $160. He bought a shirt, a pair of pants and some socks. He spent half of his money on pants. He spent ⅛ of his money on socks. He spent the rest of his money on a shirt. How much did he spend on each item?

Figure 3.4

Compare Difference Unknown	**Compare Bigger Part Unknown**	**Compare Smaller Part Unknown**	**Multiplicative Compare**	**More Multiplicative Comparison**
Sue had $567. Luke had $560. How much more money did Sue have than Luke? Or How much less money did Luke have than Sue? How much do they have altogether?	Kay has $245. Lucy has $20 more than her. How much money does Lucy have? How much do they have altogether? Or Kay has $245. She has $20 less than Lucy. How much does Lucy have? How much do they have altogether?	Sue had $789. She had $79 more than Luke. How much did Luke have? How much do they have altogether? Or Sue had $789. Luke had $79 less than she did. How much did he have? How much do they have altogether?	*Bigger Unknown* Maribel has $55. Nancy has 3 times as much as she does. How much does Nancy have? How much do they have altogether? *Smaller Part Unknown* Nancy has $165. She has 3 times as much as Maribel. How much does Maribel have?	*Difference Unknown* Nancy has $165. Maribel has $55. *How many times as much money does Nancy have as Maribel?* How much do they have altogether?

Figure 3.5



Mr. Lou had a party. He went to the bakery and bought several pieces of different types of pies. Each piece cost $2.75. He had 10 dollars. How many pieces of pie could he buy? What did he spend? Did he have any money left over?

Estimate:
Explain how you estimated:

Calculate:

Money Word Problem Types

Figure 3.6 Money Problems

Students are expected to be able to solve word problems about money with $ and ¢. They can do this with either a number line or drawings of the money. *For example: Sue had 1 quarter and 2 dimes. How much money did she have altogether?*

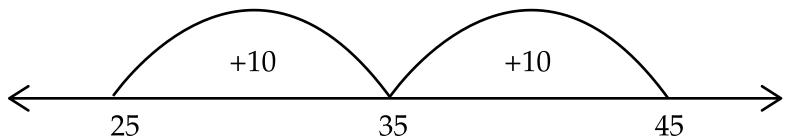
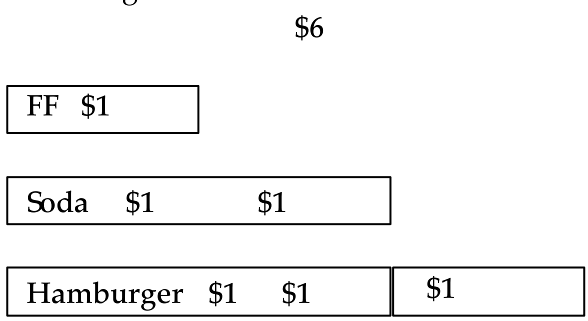


Figure 3.7 Bar Diagram Example

A soda, hamburger and French fries cost $6. The soda cost $1 more than the French fries. The hamburger was $1 more than the soda. What was the cost of each item?
Model with a bar diagram:



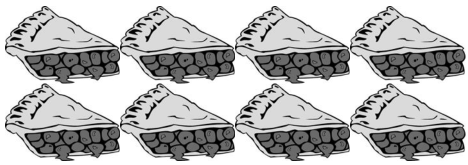
FF = $1

S = $2

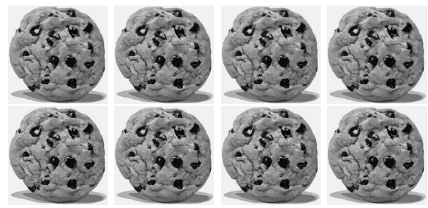
H = $3

Figure 3.8

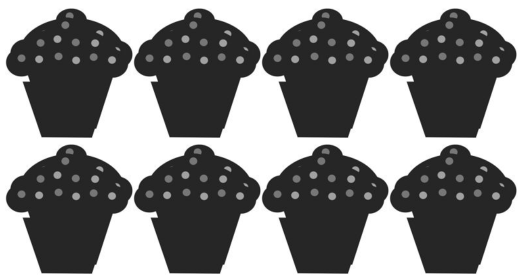
The bakery is selling delicious treats. They are selling pies for $5 each, cookies for $3 each and cupcakes for $2 each.



$5 each



$3 each



$2 each

1. What is the most expensive treat?
2. What is the least expensive treat?
3. Which costs more, 3 pieces of pie or 4 cookies?
4. If Kate bought 1 of each treat, how much would she spend?
5. If Luke bought 2 pieces of pie and Kate bought 4 cookies, who spent more money? How much more money?
6. Tim bought 14 cupcakes. Matt bought 2 fewer cupcakes than Tim. Kate bought 4 more cupcakes than Tim. Make a bar diagram to show this story. How many cupcakes did each person buy? How many did they buy altogether? How much money did they spend?

Elapsed Time Problems

Elapsed time problems are difficult for students. But, they don't have to be. First, we need to teach students to identify what they are looking for (see Figures 3.9, 3.10, 3.11, 3.12, 3.13, 3.14 and 3.15). In an elapsed time problem, we are looking for the start time, the middle time or the end time. As with all word problems, it is important for students to first realize what they are looking for.

Figure 3.9

Result Unknown Going Forward in Time	**Change Unknown Elapsed Time**	**Start Unknown Going Back in Time**	**Multistep Elapsed Time**
Tom left school at 3:10. He got home 35 minutes later. What time did he get home? Kelly ran a race in 4 hours and 12 minutes. If she started at 7:15, what time did she finish? Grandma Betsy is baking pies. The pies will take 52 minutes to cook. She put them in at 7:18. What time should she take them out?	Ray went to play basketball at 12:30. He played until 4:49. How long did he play? Farmer Tim went to bed at 11 pm. He got up at 6 am with the chickens. How long did he sleep? Grandma Betsy is baking pies. She put them in at 4:23. She took them out at 5:15. How long did she bake the pies?	Susie went to the mall. She got there at 4:45. She left her house 15 minutes before she got to the mall. What time did she leave her house? Susie has to be at the movies at 6:30. It will take her 15 minutes to get there. What time should she leave the mall? Grandma Betsy is baking pies. It is now 5:15. She baked the pies for 40 minutes. What time did she put the pies in the oven?	The early movie and the late movie last the same amount of time. The early show begins at 11:55 am. and ends at 1:37 pm. The late show starts at 9:05 pm. At what time does the last movie end? Show your work. **This problem is adapted from the fourth grade 2015 NAEP.*
Start time | Min. & hours later | End time ?????	Start time | Stop time | Elapsed time ????? (how much time passed)	Start time ??? | Min. & hours before | End time ????	

In order to get students to recognize these types of problems, do a variety of things:

1. Make workstation activities with bags for each of these types of problems so that students can practice them and improve.
2. Make workstation activities with mixed problems where the students have to sort the problems under the correct label by type.
3. Give students multi-choice, fill-in-the-blank and extended response for workstation activity sheets as well as homework.

Figure 3.10

The kids went to the movies. It lasted for 1 hour and 47 minutes. They went at 2:10 pm. What time did they get out?

a. 3:00
b. 2:47
c. 2:57
d. 2:10
e. None of the above

The kids went to the movies. It lasted from 2:30 until 4:00. How long did it last?

a. 2 hours and a half
b. 4 hours
c. 1 hours and a half
d. None of the above

Figure 3.11

Start Time	End Time	Elapsed Time
5:30	6:25	
	9:15	45 minutes
1:15		34 minutes

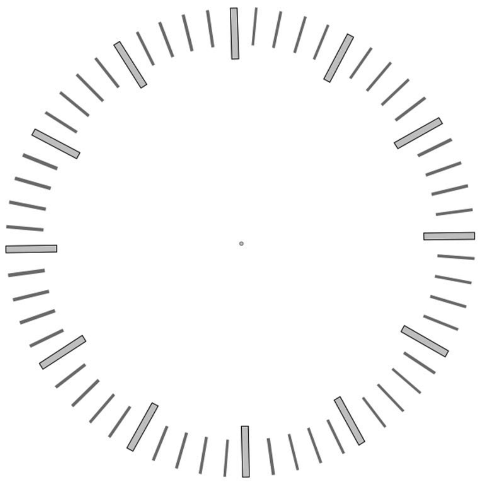
Figure 3.12

Tom spent 2 hours and 50 minutes fishing. He finished at 5.

Part A: Write the time
What time did he start?

____

Part B. Draw his start time on the clock.



Part C: Explain your thinking. Use numbers, words and a model (table, number line or the diagram (mountains, rocks . . .).

Clip art from http://www.clker.com/clipart-clicker.html

If students use the different models and strategies to solve these problems, they become more proficient. Tables, diagrams and number lines all help students to break down the problem.

Tables/T-charts

Kelly left her house at 1:15. She was gone for 2 hours and 25 minutes. What time did she come back?

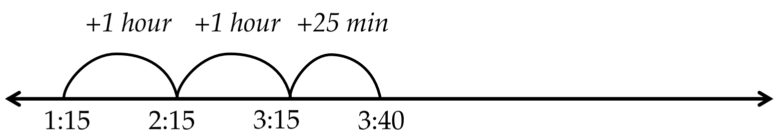
Figure 3.13

Time	Hour/Minutes
Start at	
1:15	+ 1 hour
2:15	+ 1 hour
3:15	+ 15 minutes
3:30	+ 10 minutes
3:40	Total of 2 hours and 25 minutes

Number Lines

Kelly left her house at 1:15. She was gone for 2 hours and 25 minutes. What time did she come back?

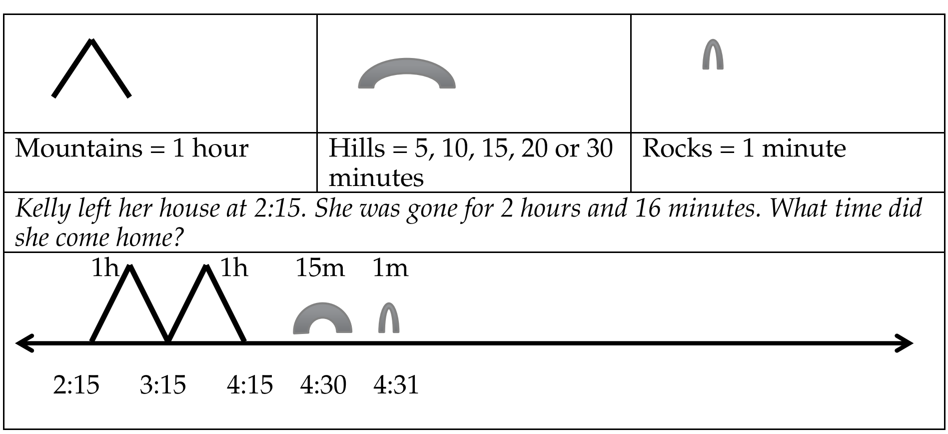
Figure 3.14



Mountains, Hills and Rocks

This interesting strategy can help students navigate the open number line. It can be very helpful because it gives them a framework for not only representing but also distinguishing the hours, the 5-minute increments and the 1-minute increments.

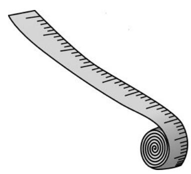
Figure 3.15



Measurement

Measurement problems involve knowing different units in the customary and metric system. Students have to understand length, mass, capacity, time and distance by the end of fifth grade. Here are a few of the problem types with some suggestions on teaching them (see Figures 3.16 and 3.17). First and probably most important and least done, is that when we are teaching measurement problems, we should actually have students measure. They should use real material or paper to do the actual measurement and cutting. Look at the following problems. For the first problem, I would give the students a 15-inch piece of material and have them prove it by acting out the problem. Then I would have them draw their model. For the second problem I would give them 2 feet of butcher paper and have them act out the problem by measuring and cutting the paper. These kinds of activities give students concrete experiences so that they can relate to the problem. I find that we rarely do this, but if we did, they would understand basic units and conversion problems so much better.

Figure 3.16



Marta was going to make tablecloths for her little cousins' doll tables. She needed to cut 2-inch pieces from a 15-inch piece of material. How many can she make? Will she have any left over? Can you use the tape measure to help her?

Figure 3.17

Claire had 2 feet of material. She needed to cut five 6-inch pieces. Does she have enough material? If not, how much more does she need?

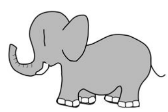
Next is a problem about mass (see Figure 3.18). Students have no idea about mass because we don't discuss it in our daily lives. They are more likely to know about units of measurement in the customary system (just barely). Have the students act out this problem on the scale. There is *no way* that you should be doing a *measurement unit* **without a scale.** Students should also think about what is the best way to figure this out. Here a table would be great!

Figure 3.18

At the candy store, they sold 500 grams of chocolate for $2. Marcos bought 3 kilos of chocolate for a party. How much money did he spend?

Here is a great problem that gets students to reason about mass (see Figure 3.19). One way to get students to reason about mass and weight is to bring in objects and weigh them and discuss the units. Also, have students cut out pictures from magazines, newspapers and circulars and have them write about those items. Have students write photo essays where they have to discuss the measurements of animals or whatever interests them. One thing about measurement problems is that students need benchmarks. So, you should talk about some of the major benchmarks, like a math textbook weighs a kilo, an average adult weighs around 100 kilos or a paper clip weighs about a gram.

Figure 3.19

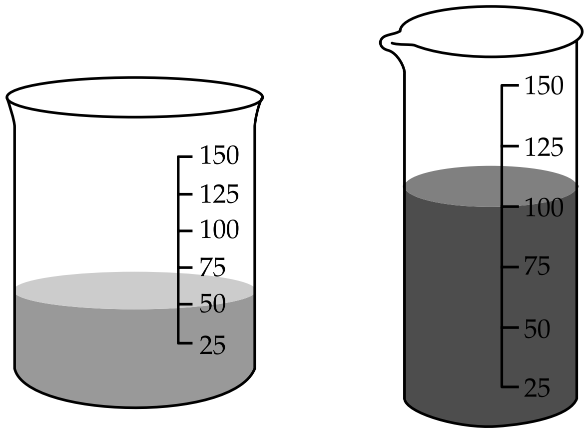


Joe and Kylie were estimating the weight of the baby elephant that they saw at the zoo. Joe said that it weighed about 100 kilos. Kylie said she thinks it weighs about 200 grams. Who do you think has the best estimate, and why?

More Measurement Problems, Drawings and Sketches

When working with capacity problems, students should actually measure out amounts. They should use and then make drawings of cups, bottles and beakers to solve problems (see Figure 3.20). The problems should involve the different operations and one or two steps. Students own the understanding of these problems when they do them.

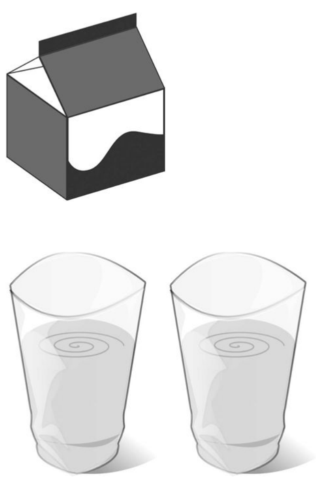
Figure 3.20



Here is another example of a problem where the picture can scaffold the thinking. Students should also act out the problem (see Figure 3.21).

Figure 3.21

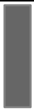
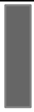
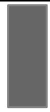
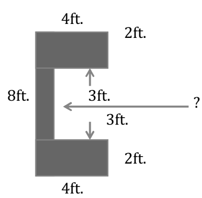
Grandma is serving fruit punch. She needs enough for ten 8-ounce glasses for each grandchild. How many pints of fruit punch should she buy?



Perimeter Problems

Area and perimeter problems can get complicated for students when there are unknown sides. Also, students tend to get confused when we start looking at rectangles with the same perimeter and different areas or with the same area and different perimeters (see Figure 3.22).

Figure 3.22 Perimeter Problems

Finding the perimeter of a rectangle	**Finding the missing side of a rectangle**	**Finding the area of a rectangle**	**Finding the area of a square**	**Finding the perimeter of a square**
				
Tammy's table is 4 m by 2 m. What is the perimeter? What is the area?	Tammy's table has an area of 8 square feet. It's length is 4 feet. How long is its width? Tammy's rectangle swimming pool was 12 meters wide. It covered an area of 84 square meters. What is the length of the pool? The perimeter of a square is 800 cm? How long is each side?	Tammy's table is 4 m by 2 m. What is the area? Tammy's kitchen is 10 meters long and 9 meters wide. What is its area?	Tammy's table is a square. One of the sides is 2 m. What is the area of the table? A square shed has sides that are 4 feet long. What is the shed's area?	Tammy's square table has an area of 16 square meters. What are the sides?
Open–ended perimeter problems	Finding the perimeter of an irregular polygon	Perimeter decimal problems	Perimeter with multiplicative comparison	Perimeter with comparative elements
Farmer John planted a garden with an area of 30 square feet. What could the possible dimensions of the fence be?	Tammy has a U-shaped table. What is the length of the inner side? What is the perimeter of the entire figure?  	Granny's garden is 1.5 times as long as it is wide. It is 5 feet wide. How long is it? What is the perimeter? What is the area?	A small rectangle garden is twice as long as it is wide. If the area is 8 square feet, find its dimensions.	The width of a small rectangle garden is 3 inches less than its length. If the area of the rectangle is 28 square inches, what is its width?

Distance Problems

I think it is really important to teach students how to think about distance problems. It is in all the standards, but I never see it emphasized in the actual teaching of measurement problems. However, this is one of the most typical situations that we do in life. People calculate distance all the time. It is relevant, familiar and important. There should be map problems across the year (see Figure 3.23).

Figure 3.23 A Trip to Colorado

Driving from Los Angeles, California, to Denver, Colorado, takes 15 hours and 18 minutes. It is about 1,018 miles, or 1,638 km, to drive one way. The Smiths are taking a summer vacation. They are going to drive from Los Angeles to Denver in 4 days. They want to drive about the same number of hours each day.

Questions

1. About how many hours will they drive each day?
2. How many miles will they travel altogether round trip?
3. What if they chose to drive in 2 days instead of 4, how long would they drive each day then?
4. How many miles would they drive each day if they drove ½ way each day?
5. Movies are about 1 ½ to 2 hours. If the kids want to take enough videos to last the whole trip, about how many should they take? What are some possible combinations?
6. The car goes 32 miles per gallon. How many gallons will they need for the trip?
7. It takes 22 gallons to fill up the car. How many times will they have to fill up the car?

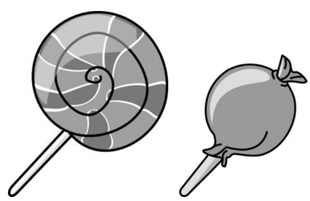
8. Gas cost $2.29 a gallon. About how much will they spend on the trip?

Adapted from http://www.mathsproblemsolving.com/number-problem-solving.html

Fraction Problems

One of our main jobs throughout the upper elementary grades is to get our students on friendly terms with fractions. This seems like a somewhat daunting task, but it can be done. The way to do it is throughout the year. Students don't learn how to do fraction problems during the unit on fractions. They learn how to do fraction problems throughout the year by seeing them often and using a variety of tools, models and strategies to think about the problems. Take the example below. This is an adaptation of the often cited Van de Walle (2007) problem where the students have to reason about the size of the whole (see Figure 3.24). Think about how the use of a picture prompt helps students to navigate through the problem. I would give the problem to the students and let them discuss it and then later show the picture prompt to see if anyone changes their thinking.

Figure 3.24



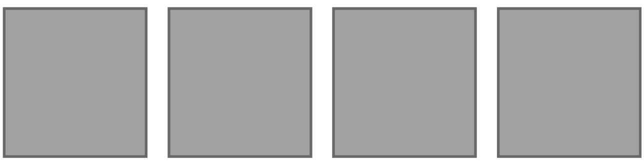
Jamal said that he ate ⅓ of his lollipop. Tim said he ate ½ of his lollipop. Jamal said he ate more. Tim said he ate more because ½ is bigger than ⅓. Jamal said that is not always true. Who is correct? How do you know?

Explain:

In the problem below, students are asked to think about half in different ways (see Figure 3.25). Students usually have very narrow ways of thinking about numbers. So, it is important to get them to think about fractions as the amount of the whole and not as something that is the same size and the same shape. So half can look lots of different ways. There is a great activity on PBS Cyberchase (http://pbskids.org/cyberchase/media/games/fractions/fractions_intro2.html) and Mathwire (http://mathwire.com/problemsolving/thirteenways.pdf) called *Thirteen ways of looking at half*, where students are challenged to find 13 ways to make a half. I would have the students actually cut the sandwiches because this is different than having them simply draw it. They own it in a different way through physically cutting the paper.

Figure 3.25

Grandma Betsy made her 2 grandchildren a turkey sandwich. Show 4 ways that she could have cut the sandwich so they each got an equal amount.



Decimals

Decimals are scary for students and teachers. Money models are good but definitely shouldn't be the only thing we use to teach decimals. Students should use wheels, bars and grids to think about decimals. Think about this problem (see Figure 3.26). What would be a good model for students to use to solve this (to show their thinking)? Have the students reason about the problem before they solve it. A common error would be for students to say that 0.125 is bigger than 0.25. So getting students to reason with tools and models helps to ground their understanding.

Figure 3.26

Jane went to the candy store. She bought 5 bags of sweet candy that weighed 0.25 of a pound and 5 bags of sour candy that weighed 0.125 of a pound.

Which type of candy did she buy more of?

How much more of that candy did she buy?

Figure 3.27 Decimal Grid

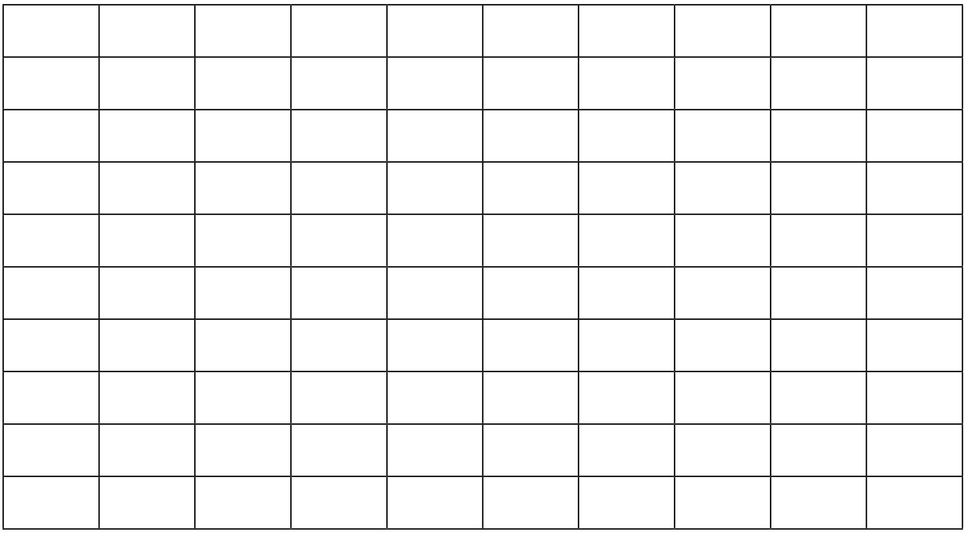


Figure 3.28

Decimal Squares®

These are a great manipulative created by Bennet (2009). He has hands-on manipulatives, books and interactive games. Check them out at http://www.decimalsquares.com/.

There are also many sites that use the decimal grids—just Google "printable decimal grids." You can download ones, tenths, hundredths and thousandths.

It is important to use decimal grids and not base ten blocks when teaching decimals because using base ten blocks can confuse students (see Figures 3.27 and 3.28). They have already learned it as a representative of one system, and then suddenly they have to rename everything. Using number grids allows students to work with a new model. The next problem is another example of getting students to reason about decimals. In this problem students are asked to think about the thinking of others, which is an excellent strategy and one that students should do often. Students need to know how to follow the logic of others and determine whether or not it makes sense and be able to speak about their thought process (see Figure 3.29).

Figure 3.29

> ### Sample Problem
>
> Riki said that 7 hundredths is more than 2 tenths because 7 is more than 2. Jessie said that is not true because tenths are more than hundredths. Use the decimal grids to reason out the problem.
>
> Explain your thinking.

In the next problem, students are expected to actually measure out the problem. By acting it out, the problem begins to make more sense (see Figure 3.30).

Figure 3.30

> The wingspans of birds can be amazing. The wandering albatross has a wingspan between 2.51 and 3.5 meters! This is the largest living bird.
>
> Measure the least and greatest sizes.
>
> Sketch it out on butcher paper.
>
> The museum wants to make a display case of the wandering albatross. If the wall is 15 feet long, about how many albatrosses can they show spreading out their wings to full wingspan at one time?

Key Points

- There are many different problem types across the domains.
- Operations and algebraic thinking problems focus on single-digit and multi-digit multistep operations.

- Measurement problems focus on a variety of topics including money, time, area and perimeter, mass, capacity, conversions, angle and volume.
- Fraction problems should be acted out.
- Decimal problems should use wheels, grids and bars.

Summary

There are many different types of word problems. Most states teach similar topics in similar grades. It is important to know the word problem landscape for the current grade that you teach as well as the other grades—partly, because word problem knowledge is cumulative. It is important at the beginning of the year to assess the word problem knowledge from the prior year to ensure that everybody is ready for the grade-level problems. Think about the many different scaffolds that can be used to help students access the problems. Also, think about the importance of students being able to understand the types of problems—across the domains. Make sure that there are posters and anchor charts up so that students can refer to them and use them to analyze the problems they are working on.

Reflection Questions

1. How do you organize the teaching of word problems? Do you frame them in any kind of way currently?
2. When teaching elapsed time, do you frame the types of problems and get students to focus on what they are looking for and the model that they want to use before just jumping in to find the solution?
3. When teaching word problems, do you scaffold understanding through picture prompts?
4. What are two or three big takeaways from this chapter? What will you do tomorrow?

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4

Structures to Scaffold Success

Posters play an essential role in classrooms. They centralize information and provide instant access to frameworks for working. Students should see strategies, models, rubrics and checklists posted to facilitate their thinking.

There are a variety of structures that can scaffold success for problem solving. These include strategy banks, model banks, rubrics and checklists. These structures provide resources for thinking about strategies, models and the process of problem solving. They provide a way for reflecting on the work in a methodical manner. These structures provide mental and concrete scaffolds for word problem work. It is one thing to say to a student, "Check your work," while it is another to say, "Here is a checklist, look it over and use it to think about how you solved this problem."

Strategy Banks

Strategies are ways of working with numbers. Strategies are the stuff you do with the numbers, the ways in which you think about adding or subtracting them. A strategy bank can be housed on an anchor chart, a student individual resource sheet or something written on the word problem itself (see Figures 4.1, 4.2, 4.3, 4.4, 4.5, 4.6, 4.7 and 4.8). Figure 4.1 is an example of what that could look like.

You want to get to the point where you can list the strategies and the students know what they are. These can be listed at first on the tests so students can see the strategies and choose one way and check another.

Figure 4.1 Four Strategies for Adding

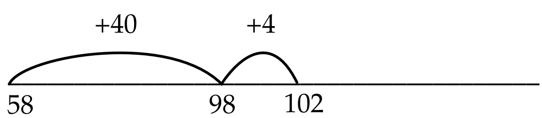
Partitioning 58 = 50 + 8 +44 = 40 + 4 102 = 90 + 12	Traditional Method (Grandma's Way) 1 29 +77 106
Using Friendly Numbers (on the number line model) 	Compensation 398 + 555 400 + 553 = 953

Figure 4.2 Four Strategies for Subtracting

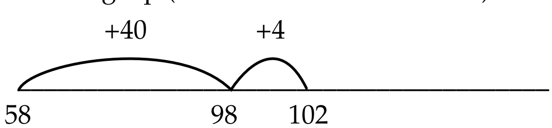
Partial Differences 102 – 58 = 44 102 – 2 = 100 100 – 56 = 44	Traditional Method (Grandma's Way) 9 12 1~~0~~2 *–58* 44
Counting Up (on the number line model) 	Partitioning 102 – 58 90 102 = 1~~0~~0 + 12 –58 = –50 + 8 40 + 4

Figure 4.3 Strategies for Multiplying

Distributive Property (modeled with open array) 15×18 $10 + 8$ 10 | 100 | 80 + 5 | 50 | 40 $100 + 80 + 50 + 40 = 270$	Doubling and Halving 15×18 $30 \times 9 = 270$
Partial Products (modeled with carets) 15×18 /\ /\ 10 5 10 8 $10 \times 10 = 100$ $10 \times 8 = 80$ $5 \times 10 = 50$ $5 \times 8 = 40$ 270	Paper and Pencil (Place value) 4 15 × 18 120 150 270

Figure 4.4 Strategies for Dividing

Partial Quotients (modeled with open array) $589 \div 7$ 40 + 40 + 4 r1 280 | 280 | 28	Halving and Halving $144 \div 12$ $72 \div 6$ $36 \div 3$ 12
Paper and Pencil (Grandma's Way) 8 4 7 | 5 8 9 –5 6 2 9 –2 8 1	Friendly Dividends $\frac{589}{7}$ 40 + 40 + 4 $\frac{280 + 280 + 29}{7}$ r1 84 r1

Figure 4.5 Addition Strategy Bank

Ten and More
Looking for Lucky 8 or 9
Friendly Numbers Counting Up
Paper and Pencil (Grandma's Way)
Break Apart Partial Sums
Give and Take
Splitting Tens and Ones

Figure 4.6 Subtraction Strategy Bank

Partial Differences

Add to Subtract (Count Up)

Paper and Pencil (Grandma's Way)

Same Difference

Figure 4.7 Multiplication Strategy Bank

Friendly Numbers

Repeated Addition

Paper and Pencil (Grandma's Way)

Break Apart

Partial Products

Distributive Property

Figure 4.8 Division Strategy Bank

Partial Quotients

Repeated Subtraction

Halving and Halving

Paper and Pencil (Grandma's Way)

Think Multiplication

Model Banks

There is a difference between a strategy and a model. A strategy is a way of thinking about the numbers and doing the math. A model is a way of showing how to solve the problem. Models include drawings/sketches, number lines, tables and tape diagrams. Models can also include using the 5 or 10 frame, the open number line and 100 grids. A model bank can be housed on an anchor chart, a student individual resource sheet, or something written on the word problem itself. Here is an example of what that could look like (see Figures 4.9, 4.10, 4.11, 4.12 and 4.13)

Figure 4.9 Models We Can Use to Solve Problems

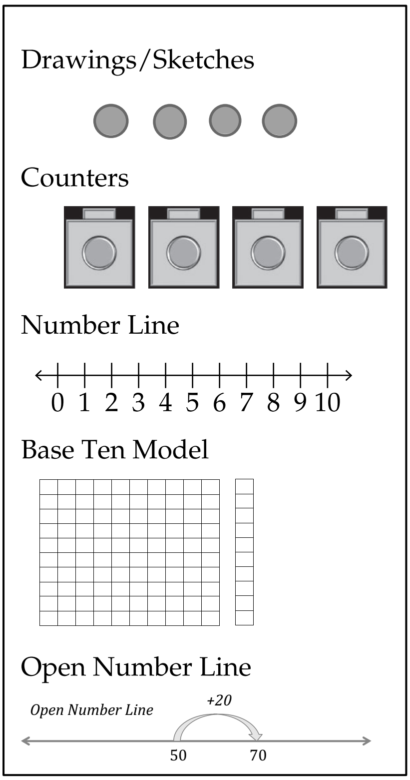


Figure 4.10 Models We Can Use to Solve Problems

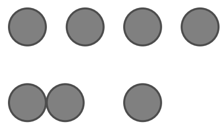
1. **Manipulatives (bears, counters)**
2. **Drawings/Sketches**
3. **Tables**
4. **Diagrams**

Figure 4.11 Problem Solving Idea Mat

Problem Solving Idea Mat

Visualize it!

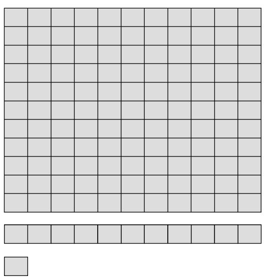
Sketch it!



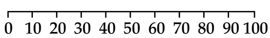
Grid it!

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Base Ten it!



Number Line it!

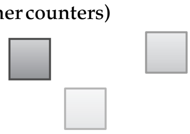


Math Bar it!

Equation it!

$4 \times 5 = 20$
$9 \div 3 = 3$
$127 - 49 = 128 - 50 = 78$
$187 + 59 = 186 + 60 = 244$

Manipulate it!
(Use the bears, the cubes or other counters)



Rekenrek it!

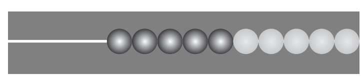


Photo: Dollar Photo

Rekenrek: http://www.mathlearningcenter.org/web-apps/number-rack/

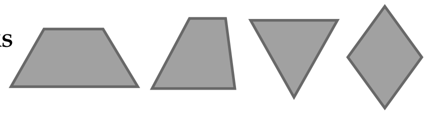
Adapted from Dusd: http://www.dusd.net/cgi/files/2013/10/Unpacking-Menu.pdf

(Newton Education Solutions, 2016)

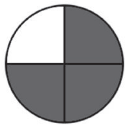
Figure 4.12 Fraction Models

3 ways to model fractions: What are you going to use?

1. Pattern blocks



2. Drawings



3. Number line

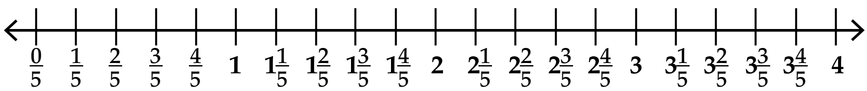
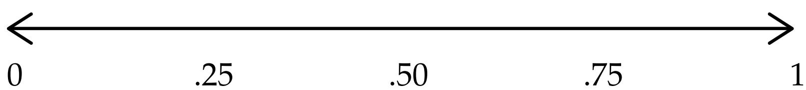


Figure 4.13 Scaffolding Thinking

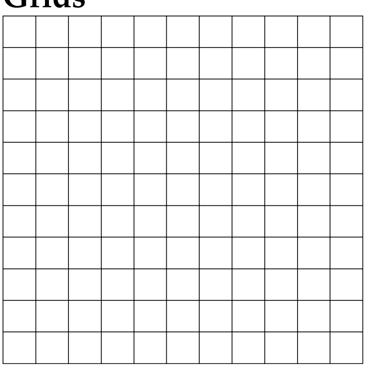
3 different ways to model decimals: What are you going to use?

Number lines



Money

Grids



Here is an example of what it might look like in a template (see Figure 4.14):

Figure 4.14 Template Example

The toy store had 15 boxes of marbles. They had 18 marbles in each box. How many marbles did they have altogether?
Visualize: Make a picture in your head.
Summarize: Talk it out. What is this story about? What are you looking for?
Set-up equation: Be sure to put a symbol for the part you are looking for.
Plan: How will you solve it? (Possible models: math sketch, open array, area model)
Model your thinking.
Solve with numbers: What strategy could you use? (Possible strategies: doubling and halving, traditional method, break apart, compensation)
Check the math.
Check the answer: Does it make sense?
Answer: ______________

Here is another example of how to scaffold the thinking behind problem solving (see Figure 4.15). In this example, the students read the problem and then think about the questions. As they decide the different solutions they will use, they can refer to the model bank. These are the different models that they have learned throughout the unit.

Figure 4.15 Scaffolding Thinking

Name ____________________

Problem Type: Equal Groups

PROBLEM: Mr. Thomas ordered 4 pizzas.
Each pizza was cut into 8 slices.
How many slices of pizza did he get?

SOLUTION 1

THINK ABOUT IT
What do you know?
What is the unknown?
What do you need to do?
Which strategies can you use?

SOLUTION 2
Solve the problem a different way

MODEL BANK
Dots and circles
Make a model
Open number line
Draw an array
Write an equation
Bar diagram

Explain

(Newton Education Solutions, 2016)

Checklists

Checklists are important because they help students to monitor their work. They help students to engage in "*thinking about thinking*" and their work (Livingston, 1997). Checklists allow students to reflect on their work, evaluate what they have done and then change anything if necessary. These skills are necessary and must be intentionally scaffolded in the learning process. Checklists and rubrics allow students to self-assess and then self-correct if needed throughout the learning process. The research shows that metacognitive skills can be taught to students to improve their learning (Nietfeld & Shraw, 2002; Thiede, Anderson & Therriault, 2003). A checklist provides a metacognitive scaffold (see Figures 4.17 and 4.18). The hope is that students use them enough so that they are internalized and students can eventually begin to do this process on their own without the scaffold.

In this example of a checklist, we see a column for self-checking, peer checking and teacher checking (see Figure 4.16). The self-check has some rows that the other two do not have because only the actual problem solver would know these two.

Figure 4.16 Word Problem Checklist

	Self	**Peer**	**Teacher**
Did I read the problem twice?			
Did I make a picture in my head?			
Did I write a set-up equation?			
Did I make a plan?			
Did I model my thinking			
Did I write my solution equation?			
Does my answer make sense?			

(Newton Education Solutions, 2016)

Figure 4.17 Word Problem Checklist

Name ________________________ Date ________________

1. Did I visualize this problem and summarize this problem?
   Yes No

2. Did I make a written plan?
   Yes No

3. Did I write the set-up equation?
   Yes No

4. Did I solve one way?
   (counters, number frame, drawings, diagram, table)
   Yes No

5. Did I check another way?
   (counters, number frame, drawings, diagram, table)
   Yes No

6. Did I write the solution equation? Does it make sense?
   Yes No

7. Did I explain what I did? Did I talk about the models I used?
   Did I talk about the strategies I used? Did I use math words?
   Yes No

(Newton Education Solutions, 2016)

Figure 4.18 Scaffolding Thinking

My Word Problem Checklist

- [ ] I will visualize the problem.
- [ ] I will summarize the problem.
- [ ] I will make a plan.
- [ ] I will solve one way.
- [ ] I will check another.
- [ ] I will think about if the answer makes sense.
- [ ] I will explain my thinking.

(Newton Education Solutions, 2016)

Rubrics

Rubrics are another way for students to check their work and the work of others. They should be made with the students. Rubrics should give students a clear picture of what they are supposed to be doing. A rubric should be the springboard for an ongoing conversation about the math. What is being done well and what needs to be improved and what are the immediate next steps. For example, if a student gets a 1, we discuss what they need to get a 2. Even the big kids like the ice cream rubric and they totally get it. They understand, "Well I have 2 scoops, and I need another scoop. My plan isn't clear enough. I only got some of the answer correct. So, I am going to go back and work on my plan first."

Rubrics list all of the things that are being counted for that work (see Figures 4.19, 4.20, 4.21 and 4.22). So students know and understand exactly what it is they have to do. They should be given the rubrics before they are given the task. I highly encourage teachers to use these rubrics as examples (always make them student friendly because the point is that students use them). Also, and just as important, align your rubrics to your state testing rubrics so that students are honing the required skills throughout the year.

Figure 4.19 Word Problem Rubric

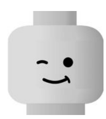
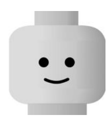
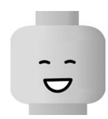
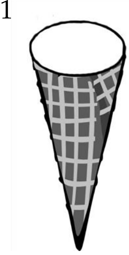
Student's Name: **Date:**					
	Need Work  **4**	**Ok**  **3**	**Good**  **2**	**Great**  **1**	**Score**
Make a plan (stated the plan)					
Draw a model					
Write a set-up equation					
Pick a strategy to do the math					
Explain your thinking					
Write a solution equation and write the answer with the units					

Figure 4.20 Problem-Solving Rubric: Go for 3 Scoops

Score	**Make a plan State the plan**	**Solve one way Check another correct answer**	**Write a set-up equation and a solution equation**	**Detailed, clear explanation**
4 	Excellent, clearly stated plan	Solves one way and checks another; uses two sophisticated ways; has the correct answer	Writes both a correct set-up equation and a correct solution equation	Explains in detail both the strategies and the models; connects back to the original plan and walks us through the solution
3 	Good plan made clear	Solves one way and checks another; has the correct answer	Writes both a correct set-up equation and a correct solution equation	Gives a simple, clear, thorough explanation
2 	States some of the plan	Solves one way; has a partially correct answer	Writes only the correct solution equation	Brief explanation; might talk about one aspect of the plan
1 	No plan evident	Attempts to solve or doesn't solve the problem; does not have a correct answer	Does not write an equation or writes an incorrect equation	Does not give an explanation

(Newton Education Solutions, 2016)

Figure 4.21 Problem-Solving Rubric

Name: ________________________ Date: _________________		
Criteria	**Possible Points**	**Points Earned**
Read the problem/ Underline the question	**1**	
Make a plan	**1**	
Set-up equation	**1**	
Solved one way	**1**	
Checked another	**1**	
Correct solution equation	**2**	
Answer labeled/Check the answer against the question	**1**	
Explanation	**2**	
Total	**10**	

Figure 4.22 Word Problem Rubric

0	**No understanding**	**Blank paper**
1	**Novice:** **None or very little understanding of the problem (below grade level)**	**Shows no or very little understanding of the problem** ***Made an attempt to solve the problem***
2	**Apprentice:** **Somewhat understands**	**Partially correctly solves the problem. An apprentice always has a partially correct answer.** ***Can have a correct drawing or a correct equation*** ***Could get started but couldn't finish the problem***
3	**Practitioner:** **Fully understands (on grade level)**	**Correctly solves the problem** ***All parts of the problem are answered correctly*** ***Uses numbers, words and pictures***
4	**Expert:** **Completely understands and goes beyond (above grade level)**	**Correctly solves the problem** ***All parts of the problem are answered correctly*** ***Uses numbers, words and pictures*** ***Shows more than one way and has an extensive explanation***

The Art of the Explanation

The explanation is important. Students have to be taught how to explain what they did to solve the problem. Practice this with students through doing think alouds. First, teach students to write about how they read the problem, decided what was missing and then made a plan. Next, teach them to discuss both their models and strategies. Finally, teach them to discuss how their answer addresses the question. This is why it is so important for students to actually follow a plan when solving the problem so that they can go back and discuss what they did in some sort of focused and organized manner (see figures 4.23, 4.24, 4.25 and 4.26).

Figure 4.23

Sue had 2 marbles. Her brother had 5 more than she did. How many did he have?

Explanation

I read the problem. It was a compare problem. I was looking for the bigger part. I added the 5 to the 2 and I got 7. I drew a picture to show my thinking. It makes sense.

Figure 4.24 Explaining Thinking

David had 5 marbles. Kenny had 2 times as many as David did. How many did they have altogether?

Explanation

I read the problem. I decided it was a compare problem. I had to multiply 2 × 5 to find the answer. I drew a bar diagram with 5 on top and 2 boxes of 5 on the bottom because it was 2 times as many. I wrote an equation of 2 × 5. Then, I added 5 + 10 together and got 15. They had 15 marbles altogether. I checked the question with my answer and it makes sense.

Starter Frames

It is helpful to give students starter frames as scaffolds. Some of the frames are as shown in Figure 4.25.

Figure 4.25 Explaining Thinking

I made a plan. I decided to ____________________ because ______________.
I checked my work by ____________________________.
I used ______________________ to model my thinking.
I solved by using the ___________________________ strategy.
I knew that my answer was correct because __________________________ _______________.

Use an explanation rubric with the students so that they know what to look for. Model writing an explanation with your problem of the week. Always have students explain their thinking in guided math groups and math workstations.

Figure 4.26 Explanation Rubric

5	4	3	2	1
I wrote exactly what I did. I talked about the models and the strategies. I explained all my steps in detail. I explained what I did and why I did it.	I explained what I did. I talked about my plan.	I explained some of what I did. I talked about the models or the strategies. I need to give more detail. My explanation was a bit confusing.	I only explained a little bit. Some of my explanation is a bit confusing.	I didn't explain my problem.

Key Points

- Strategies should be hanging up on a reference poster.
- Models should be hanging up on a reference poster, too.
- Checklists help students to review their work.

- Rubrics help students to review their work.
- Explanations are an important part of problem solving.

Summary

Problem solving is involved. Students should be consciously thinking about how they are solving problems, which strategies they are using and what models they are using. There are so many moving parts that it is important to use templates, checklists and rubrics so that students can monitor and reflect on their work as they are doing it and can think about whether their answers make sense. If students are aware of their process, then they can explain and justify what they did.

Reflection Questions

1. Do you help your students to understand the difference between strategies and models?
2. Do you have your students use checklists and rubrics to check their own work?
3. How often do you have students monitor the work of others by using checklists and rubrics?
4. What is your biggest takeaway from this unit? How will it impact your current practice?

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5

The Language of Word Problems

Things to Think About

> *For many students who struggle with mathematics, word problems are just a jumble of words and numbers.*
>
> *—Zorfass, Gray and PowerUp*

People often say language doesn't matter. But consider the fact that we say "Turn the volume up" or "Measure the volume of this box." We say, "Set that on the table" and "Let's make a table to solve that." We say, "That's odd," but ask, "Which one of these is an odd number?" We say, "I want some of those," and we ask, "What's the sum?" The language of math is so difficult. We have to consider the different types of words and how they have multiple meanings in everyday language and then in math.

One time I asked a student what an "expression" was and he said, "Oh that's something you make on your face." I said, "Yes it is, but what about in math." He had no idea. I asked another student, "What does the word multiple mean?" She said, "Oh you know it is when you have a lot of copies of something." One of the first things that I do when I am unpacking a word problem with students is to make sure that they understand what all the words in the problem mean. If we are going to help students to get better at word problems, we need to help them get better at understanding math words. Here are six things to think about concerning language and word problems.

1. Math Language Is Specialized

Researchers have written about the specialized nature of math vocabulary (Rubenstein and Thompson, 2002; Heinze, 2005; Rubenstein, 2007; Gay, 2008). They note that it can be difficult for a variety of reasons.

1. Some words are used in both everyday English and in math, but have different meanings in each context. For example: right angle versus right answer.

2. Some words are specific math words. For example: addend, minuend, quotient.
3. Some words have more than one meaning in math. For example: "A circle is round" or "Let's round 145 to the nearest ten."
4. Some words are homophones to everyday English words. For example: sum and some.
5. Some mathematical concepts are verbalized in more than one way. For example: one-quarter versus one-fourth.
6. Some words are learned in pairs that are confusing for students. For example: multiple and factor or area and perimeter.
7. Students sometimes use everyday words (informal words) instead of math words (formal). For example: "diamond" for rhombus and "corner" for vertex.
8. Modifiers matter. For example: fraction vs. improper fraction and denominator vs. common denominator.

Math has many semantic features that confuse students. Here are a few more examples (adapted from http://steinhardt.nyu.edu/scmsAdmin/uploads/004/738/NYU_PTE_Math_Module_For_ELLS_Oct_8_2009.pdf):

Synonyms: subtract, minus, take away, decrease
Homophones: weigh, way
Prepositions: divided into vs. divided by
Passive structures: Seven dogs were sold

Teachers need to explicitly teach the math vocabulary. Students need multiple opportunities to practice the words through writing, games and talking. "Math vocabulary is inextricably bound to students' conceptual understanding of mathematics" (Dunston & Tyminski, 2013, p. 40). If students don't understand the words, then they won't understand the concepts.

2. Wording Matters

Researchers have resoundingly found that language matters when it comes to understanding and solving word problems. Many researchers have found that rewording the text of word problems so that students understand the problem and the words reflect the problem structure greatly improves problem-solving success (De Corte, Verschaffel and De Win (1985) and De Corte and Verschaffel (1987)).

Wording matters. Hudson (1983) posed this problem to some children: "There are 5 birds and 3 worms. How many more birds are there than worms?" Many of the children could not solve the problem. However, when the problem was reworded, "How many birds won't get a worm?" many of the students could solve the problem. Riley, Greeno and Heller (1983) said that rewording helps students to understand the problem, and when students understand the problem they can solve it. Cummins (1991) pointed out that "the data seem to indicate that the knowledge [to solve problems] is there, but is simply is not accessed when problems are worded in certain ways (p. 267)." She argues that students who fail do so because they are "missing" or "have" inadequate mappings of verbal expressions to part-whole structures. She maintains that "rewording enhances performance."

I really find this to be true. When students know what they are looking for, they are much more likely to find it. For example: The teacher says: *Kate had 57 marbles. She got some more for her birthday. Now she has 75. How many did she get for her birthday?* A common error is for students to add 57 and 75. Students get confused by the phrase *some more*. They will often just add 57 and 75 because they are confused by the language. If the problem is reworded:

> *Kate had 57 marbles. She got some more for her birthday.* **[She had 57 and now she has 75. She got some more to make her total 75.]** *How many did she get for her birthday?*

This scaffolds the phrase *some more*. It can be further scaffolded with a language scaffold written in the equation. It helps students to have access to the concept. The scaffolding would eventually be phased out. We need students to understand the structure as 57 + $\underset{\textit{some more}}{\underline{\hspace{3cm}}}$ = 75.

This is also about syntax and phraseology. We have to scaffold student access into understanding the word problem. We must think about ways to present the problem that make sense and scaffold conceptual understanding and then next scaffold the algebraic phrases we use in these problems.

3. Function Matters

We ask students to do a lot of different things in math, including *reading* the word problem, *writing* the answer, *illustrating* the answer, *showing* their thinking, *using* their math words, *proving* the answer with different strategies, *giving* an explanation and *sharing* their thinking. These all require different types of talking and organizing of thoughts. We have to be explicit about what we are asking students to do and then teach them how to do these different things. We have to practice with students how to talk with each other about their problem solving, how to share their thinking, make predictions, think out loud, revise their thinking, give examples, defend their thinking, challenge others, give explanations and express their "fuzziness" (their confusions). When focusing in on function, teachers need to use many language frames as scaffolds.

4. Math Texts Are "Dense and Concept Loaded"

Researchers have found that the language of word problems is "dense and concept-loaded." There is a whole lot of information in a small amount of text, and therefore the text must be read carefully, thoroughly and attentively so that students comprehend what the problem is about (Heinze, 2005). This is compounded for English language learners (Basurto, 1999). Because math word problems are "dense and loaded," teachers need to teach special reading comprehension strategies to successfully understand and solve them (Winograd & Higgins, 1994/1995). These involve reading slowly and often more than once to make sure students fully understand the problem (Kang & Pham, 1995) (see Figure 5.1).

Figure 5.1

> "Students who have difficulties with reading, computation, or both are likely to encounter difficulties with word-problem solving." (Jitendra & Xin, 1997, p. 413)

Let's look at this problem, adapted from *Illustrative Mathematics* (2015):

> The fourth-grade students were studying reptiles. They went to the reptile exhibit at the zoo. The zoo keeper explained that the bearded lizard was 4 feet long. It had grown 1 foot in a year. The spotted lizard was 2 feet long. It had grown 1 foot in a year also. The teacher asked the students which lizard grew more? Some students thought that the lizards grew the same size because they each grew 1 foot. Carol said she thought that the spotted lizard grew more because it doubled its size whereas the bearded lizard only grew ¼ more of its size. Dan said he didn't understand what she was talking about. Can you explain her thinking?

This problem is dense and loaded. There are a number of concepts from the very basic to more complex. First, students need to know what length is. Then they need to understand the unit of measure of a foot. Next, they need to understand the difference between additive and multiplicative thinking. Moreover, they need to be able to follow the thinking of others and decide whether or not it makes sense, which is a process/practice. Finally, they need to be able to explain the thinking of others. Adams (1990) noted, "The greater the time and effort that a reader must invest in each individual word the slimmer the likelihood that preceding words of the phrase will be remembered when it is time to put them all together" (p. 141) (see Figure 5.2). These are serious challenges that teachers must address when teaching word problems.

Figure 5.2

> "Language proficiency appears to be a contributing factor in problem solving; student performance on word problems is generally 10–30% below that on comparable problems in numeric format." (U.S. Department of Education, 2001)

5. Word Problems Are a Distinct Genre

Several researchers have argued that math texts are a distinct genre and that students have to learn how to unpack that type of genre (Winograd & Higgins 1994/1995; Kang & Pham, 1995; Irujo, 2007). Spanos (1993) discusses how even though they are called "story problems," they lack the traditional storytelling devices (see Figure 5.3). Spanos gave this example:

> *Sam's truck weighs 4,725 pounds. The truck can carry 7,500 pounds of rocks.*
> *What is the total weight of the truck and full load?*

Spanos noted that in a real story, we would know more about Sam and his truck. Who is he and why does he drive this truck? Where is he going? In a real story we would have more of a context, perhaps some paraphrasing and repetition to help students understand the word "load." None of this is given in this decontextualized telling of Sam and his truck. And why do we care how much his truck weighs. Perhaps in a traditional short story, Sam is a trucker and has come to a weigh center that says trucks carrying a load of more than 10,000 pounds get fined. So, when the truck gets weighed, they have to deduct the truck weight from the total weight to determine the load. This would give some context to the "story problem" situation.

Figure 5.3

> "Major concerns about basal programs include a lack of adequate provision for practice and review, inadequate sequencing of problems, and an absence of strategy teaching and step-by-step procedures for teaching word problem solving." (Wilson & Sindelar, 1991, p. 512)

Barwell (2011) (citing Gerofsky) points out how story problems have a three-part structure.

1. Scenario (set-up)
2. Information about the situation
3. Question

Barwell points out that with word problems you can use the same scenario but change the information and be able to work on different mathematical concepts. We see this concept clearly with problems like:

A. The bakery has 50 cookies. There are 10 equal rows. How many cookies are in each row?
B. The bakery has 50 cookies. There are 5 cookies in each row. How many rows?

In one problem we are looking for how many are in each group, and in the other problem we are looking at how many groups. The scenario stays the same but the information in the scenario changes and so we get at different aspects of division.

Barwell (2011) also says that word problems get so convoluted that students can overthink them and stop reasoning. He gave this problem as an example:

Steven earns $5 for every bundle of newspapers he delivers. He wants to buy a game that costs $18. How many bundles of newspapers does Steven need to deliver to earn enough money to buy this game?

Barwell (2011) says that students get distracted with thoughts like: "Who is Steven?" "How many newspapers are there in a bundle" or "What is the game he wants to buy and why is it so cheap?" He points out, "Often, students combine the numbers in the problem in apparently non-sensical ways or give unrealistic solutions. For example, students might find that Steven must deliver 3.6 bundles, rather than a more realistic four bundles." He points out students can get so bogged down in the language of the problem that there is a "suspension of sensemaking (Verschaffel, Greer & de Corte, 2000)." Sometimes, students are victims of bad word problems that are not realistic but "styled representations of hypothetical experiences (Lave, 1992)."

Furthermore, word problems use different tenses that can be very challenging. For example:

Susie is 2 times older than her brother. She will be 15 in 1 year. How old is her brother?

Students have to understand the present tense and the future tense.

6. ELLs and Math as a Language

When working with English Language Learners (ELLs) you should always have a content goal and a language goal (Echevarria, Vogt & Short, 2000). You want to always think about the ways in which you are exposing the students to the vocabulary over time.

A. Content Goal
The content goal is the math that students are working on. This should be written in an "I can" statement on the task.

B. Language Goal
When working with ELLs, (however, I would argue that this is a good practice when working with all learners), teachers should have a variety of strategies to make the language accessible to the students. Language frames are part of the tools used to scaffold access to the words and phrases. So, statements like "I am proving my thinking by . . ." should be put on a sentence frame for students to use. Another example is "I have _________ *more than* __________. I have __________ *less than* __________.

ELLs also do not necessarily have the background knowledge to comprehend math word problems (Short & Spanos, 1989; Barwell, 2011). There are various cultural differences that also hinder the understanding of word problems. Word problems that use the customary system of measurement (used only in the U.S., Liberia and Myanmar) or U.S. money might confuse students. ELL students might make a mistake and give an answer in meters instead of inches, thus resulting in 3 meters instead of about 3 inches. Let's think about a weather problem involving the temperature. If students are thinking Celsius but the problem is in Fahrenheit, then there could be a big discrepancy. If the problem said that it was 80 degrees outside and everyone had on their bathing suits, students would have a difficult time imagining this scenario if they were thinking Celsius.

The issues of linguistic complexity (readability, syntax and complex vocabulary) and word problems have long been studied (Martiniello, 2008). Martiniello looked at how the linguistic complexity interferes with an ELL's comprehension of word problems. The NRC points out that "[a] test [of proficiency in a content area] cannot provide valid information about a student's knowledge or skills if a language barrier prevents the students from demonstrating what they know and can do" (cited in Martiniello, 2008, p. xx).

Key Points

- Math language is specialized.
- Wording matters.

- Function matters.
- Math texts are "dense and concept loaded."
- Math word problems are a distinct genre.
- ELLs need a content goal and a language goal.

Summary

Language is a significant part of word problems. The linguistic complexity of the word problem affects performance (Martiniello, 2008). Students need to be able to read, comprehend and translate the specialized language used in word problems. Math vocabulary is essential to comprehending and solving word problems (Stahl & Fairbanks, 1986; Blessman & Myszczak, 2001; Pierce & Fontaine, 2009). If children know the words, then they can better understand the problem. There are so many different aspects of words that they must be scaffolded for student understanding. We also need to explicitly teach the words of doing the math and solving the problems as well. Students need a variety of ways to unpack the "dense and concept loaded" word problems that they encounter in textbooks every day. They need to learn how to work through this special "genre" in order to succeed with word problems.

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6

Reasoning About Problems

Adaptive reasoning is the capacity for logical thought, reflection, explanation, and justification.

(Kilpatrick, Swafford & Findell, 2001, p. 116)

Reasoning is one of the essential elements of mathematical proficiency. It means that students will "think logically about the relationships among concepts and situations" (Kilpatrick et al., 2001, p. 129). Students need to be able to think about different ways to approach the problem, about whether or not their answers make sense, and about the ways they can verify and justify their answers. Students in elementary school should have many opportunities for "explanation and justification but also intuitive and inductive reasoning based on pattern, analogy and metaphor (Kilpatrick et al., 2001)." Students should be encouraged to use both "mental and physical representations" as "tools to think with" (Kilpatrick et al., 2001).

Kilpatrick et al. (2001) note that "with the help of representation-building experiences, children can demonstrate sophisticated reasoning abilities (p.130)." When children work in pairs and groups to prove the math by acting it out, working with manipulatives, drawings, diagrams, tables and other models, and they are encouraged to discuss their thinking and reflect on the thinking of others, they get good at it.

Contextualizing

Students should not only solve problems but also pose them. To get students to make up word problems, they need to be scaffolded by acting them out and using concrete objects, drawings and diagrams. This activity takes time and practice. It is important to give students the context at first. For example, tell the students that the answer is 5 marbles and ask them, "What is the question?" Sometimes, I tell the students what type of problem it is. This is different from just saying the answer is 5. At first, you might tell the students it's an addition problem or a subtraction problem and have them practice those. In the upper grades, also use multiplication and division. For example: *The answer is 5 marbles and it was a division problem.*

Sometimes, I'll tell the students to write a one-step or two-step problem. Other times, the students will write problems and then we will sort those problems and decide what was the operation as well as how many steps. Templates help to outline the criteria. They also lead nicely into using the rubric to check the work. The templates hold the students responsible for all the parts of writing word problems. Notice that in the templates, there is an emphasis on a set-up equation and a solution equation. Part of reasoning is that students understand how to show the problem with equations.

Equations

From first grade up in many of the standards, students are expected to be able to write an equation with a symbol for the unknown. They are also expected to compare numbers using symbols for greater than, less than and equal to. For example: *Sue had 15 cm of string. She bought 27 cm more. How much does she have now?*

$15 + 27 = ?$

Another example: *Sue has 57 cm of string. Tara has 34 cm of string. Who has more and how much more?*

$57 > 34$

Students should be able to show that comparison using symbols: $57 > 34$. It is crucial that students understand mathematical symbols and when and how to use them when explaining their reasoning. Make sure that in the word problem work, there is an emphasis on students knowing how to show what they know with numbers and symbols (see Figures 6.1 and 6.2). Students could count up or back to find the difference.

Using Graphic Organizers to Write a Word Problem

Work with structure, meaning work on a specific type of word problem. For example, you could tell the students you are going to work on division problems. Then, on the board, do a group graphic organizer of all the elements of the word problem (see Figure 6.3).

Figure 6.1 Writing a Problem

<table>
<tr><td colspan="2">Template A</td></tr>
<tr><td colspan="2">Write the problem.

Now make a picture in your head of this problem.</td></tr>
<tr><td>What type of problem is this?
Addition
Subtraction
Part-Part-Whole
Compare</td><td>What are we looking for? Write an equation with a symbol for the part that we are looking for.</td></tr>
<tr><td>Model the problem with a sketch.</td><td></td></tr>
<tr><td colspan="2">Write the equation with the missing number in it.

Answer: ______________________units</td></tr>
</table>

Figure 6.2 Writing a Problem

<table>
<tr><td colspan="2">Template B</td></tr>
<tr><td colspan="2">Write the problem.

Make a picture in your head of the problem.</td></tr>
<tr><td>What type of problem is this?
Multiplication or Division

Is it a one-step or two-step?</td><td>What are we looking for? Write an equation with a symbol for the part that we are looking for.</td></tr>
<tr><td>Model the problem with a sketch.</td><td></td></tr>
<tr><td colspan="2">Check the problem another way:</td></tr>
<tr><td colspan="2">Write the equation(s) with the missing number in it.

Answer: ______________________units</td></tr>
</table>

Figure 6.3 Writing a Problem

Scenario	Things in the scenario	What do you want to know?
Playground Classroom Cafeteria Auditorium Bus	Swings/Slides Tables Lines Seats Students Teachers	How many groups? Or How many are in each group?

Problem 1: *There are 45 students in the cafeteria. They are at 5 tables. If there is an equal amount of students at each table, how many students are at each table?*

More Routines for "*What Is the Question*?"

The teacher can hang a poster at the beginning of the week and write: *The answer is 10 marbles. Write a word problem for this answer.* Throughout the week, the students can write problems and put them on Post-it® notes to answer the problem. Teachers can also do this activity in small, guided math groups or post this as a workstation activity.

Another Version of "*What Is the Question*?"

A great deal of research has focused on getting students to actually think about the problem situation. In these problems, the student is given the word problem but not the question. The idea is that if students focus on the problem and understand the scenario, they will intuitively know what the question should be. For example, *The toy store has 50 marbles. They got a shipment of 50 more.* Here is where students are supposed to come up with the question. This is a great way to see if students are reasoning about the problem. Here is another example: *Miguel ate ½ his candy bar in the afternoon. In the evening, he ate another ¼ of his candy bar.*

2- and 3-Bean Salad Problems

A famous set of problems out of the Lawrence Hall of Science in Berkeley California looks at developing algebraic reasoning through what is called 3-bean salad problems. Many people have taken this idea and done a great deal of work with it. I like the problems because they are engaging, hands-on and rigorous. Students start out with simple problems that get progressively more difficult. See Figures 6.4 and 6.5 for examples.

The absolutely most fantastic thing about the 2- and 3-bean problems is that they scaffold nicely into a bar diagram. So, the students solve with the beans. Then they draw a picture of what they solved. Then they put a rectangle around that picture. Then they take out the beans and just put numbers and label the rectangles. Voila! A bar diagram. Of course, you don't teach all of this at once. But you scaffold into the bar diagram (see Figures 6.6 and 6.7).

Figure 6.4 2-Bean Salad Problem

> There are 2 types of beans. There are 3 times as many red beans as white beans. There are 15 red beans. How many white beans are there? How many beans are there altogether?
>
> 1. Use the beans to solve.
> 2. Draw a sketch of the beans.
> 3. Make the sketch a bar diagram.

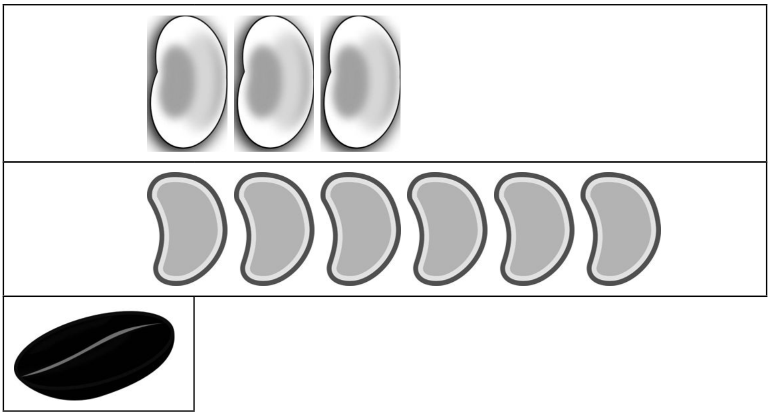
Figure 6.5 3-Bean Salad Problem

> There are 10 beans. Half of them are red beans; $\frac{1}{5}$ of them are white beans. The rest are black-eyed peas. How many beans of each are there?
>
> 1. Use the beans to solve.
> 2. Draw a sketch of the beans.
> 3. Make a bar diagram.

> Mike had some beans. Three were white beans. There were twice as many butter beans as white beans. There were 2 fewer black beans than white beans. How many of each bean was there? How many beans were there altogether?

Step 1: Act out the bean problem.
Step 2: Draw out the beans.
Step 3: Put the beans in a rectangle.

Figure 6.6 Bean Salad Problems



Step 4: Draw the rectangle with just the numbers and labels.

Figure 6.7 Bean Salad Problems

White beans	3
Butter Beans	6
Brown Beans	1

Coin Puzzle Problems

There are also Coin Puzzle problems (Lawrence Hall of Science) that elicit the same type of reasoning, but with using coins as a pretext rather than beans (see Figure 6.8). Students should be encouraged to use physical models as well as drawing, diagrams and tables to work through the possibilities.

Figure 6.8 Coin Puzzle Problems

Mike has 37 cents. He has 11 coins. What are they?

1. Draw a picture
2. Write an equation
3. Answer

Word Problem Sort

In this workstation, the students sort the word problems by category (see Figure 6.9). This station helps students to reason about the type of problem they are solving. It is important that students understand and can explain the situation. Although they might use an inverse operation or another strategy to solve the problem, they need to understand the problem situation. For example, I might use repeated addition to solve a multiplication problem, but that doesn't change the problem type. The strategy to solve the problem might have been repeated addition, but the problem is a multiplication one.

Figure 6.9 Word Problem Sort

Multiplication	Division
Mark had 7 bags with 7 marbles in each bag.	Marta had shared 12 cookies with her sister and cousin equally. How many did each girl get?

Concentration Match

There are different versions of concentration match (see Figures 6.10 and 6.11). One version is where the students match the expression to the correct problem. The other version is the students find the story that goes with an expression. Students need to be able to write the expression and/or equation that matches a problem because it shows that they can reason about the numbers—going from words to numbers. They also need to be able to reason from the numbers and be able to think about which situations match this problem (thus going from numbers to words).

Figure 6.10 Circle the Equation

Maribel had 4 bags and 5 rings in each bag. How many rings did she have altogether?

Circle all the correct expressions that represent this problem.

$5 + 5 + 5 + 5$ $\quad$ $4 + 4 + 4 + 4$

$4 + 5$ $\quad$ 4×5

Figure 6.11 Equation Match

$15 \div 3$

Circle a story that could match this expression.

a. There are 15 kids and they ate 3 apples.
b. There are 18 kids and they ate 3 apples.
c. Three kids shared 15 apples.
d. Not here.

Matrix Problems

Matrix problems scaffold logical thinking for students. They have a place to track what they are doing and think about and record all the information they are getting. We need to bring back these types of problems into our classrooms because they promote logical, step-by-step thinking with a lot of different information. It is a skill to be able to think this way, and these types of matrices build that skill (see Figure 6.12).

Figure 6.12 Reasoning Problems

Kelly and her 4 friends stopped by the bakery after studying in the library. They each bought a cupcake. Their names are Kelly, Sue, Mark, Jamal and Tom. The types of cupcakes were strawberry, vanilla, chocolate, peach and lemon.

Use these clues to tell who ate which cupcake.

Kelly only likes chocolate.
Sue does not eat fruity cupcakes.
Mark loves anything lemon.
Jamal does not eat berries.
Tom loves berries.

	Strawberry	Vanilla	Chocolate	Peach	Lemon
Kelly					
Sue					
Mark					
Jamal					
Tom					

Venn Diagrams Are Great Thinking Activities

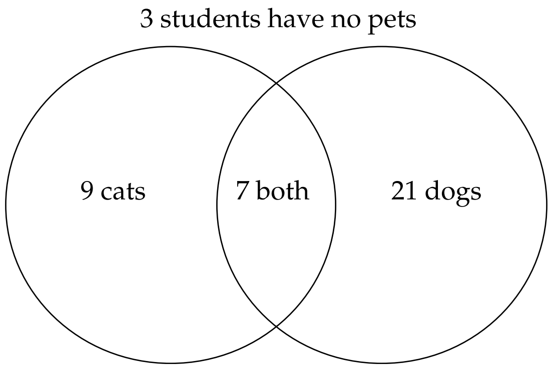
Venn diagrams also provide a way for students to think about a lot of different pieces of information. We need to also bring these back into our routines so that students learn to organize information and then think about it in clear ways. Students should start out with easy Venn diagrams, using concrete materials to act out the problems, and then move on to representational ones with pictures and then just numbers (see Figures 6.13 and 6.14). Students should also work on 3-circle Venn diagrams.

Example 1:

We surveyed 40 students in our school about their pets. Sixteen students have cats. Twenty-eight students have dogs.

1. *If 7 students have both, how many students have neither?*
2. *How many have either cats or dogs?*

Figure 6.13 Venn Diagram

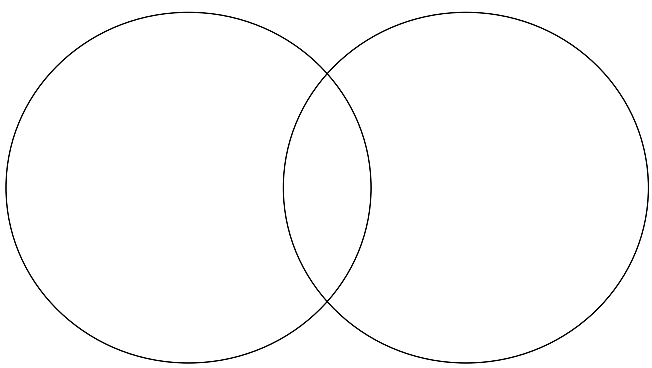


Explain Your Thinking

Well, of the 16 students with cats, 7 have both, so that means 9 have cats only. In terms of the dogs, 28 students had dogs but 7 had both, so that means 21 had only dogs. Then, 21 + 9 + 7 is a total of 37 students, so that means 3 students had no pets.

Example 2:

Figure 6.14 Venn Diagram



Kate made up a Venn diagram puzzle. She says there is a way to sort these names. Can you find a way to sort them using the 2-circle Venn diagram?

Mary Tom Lucy Mike
Macy Marvin Brenda Ned
Mark Keith Amy Zac

Explain your thinking.

Students should be able to explain that some are girls' names and some are boys' names. The overlap is boys' and girls' names that start with the letter *M*.

Table Problems

Table problems are very important (see Figures 6.15, 6.16 and 6.17). They also help students to organize information, to handle different pieces of information and to look for patterns. Students should do these problems with small numbers first, so that they can act them out, use manipulatives and then go to drawings and diagrams and eventually work completely abstractly with the table and numbers.

June and Carl built 4 snowmen. Each snowman had 5 buttons.
How many buttons did they use in all?
How many buttons will they need for 5 snowman?
If they have 45 buttons, how many snowman can they make?
If they have 12 snowmen, how many buttons will they need?

Make a table:

Figure 6.15 Table Problem

Snowman	Buttons
1	5
2	10
3	15
4	20
5	?

Another Example

Kelly has $10. She wants to buy a new game. She is saving $2 a day for the next week. How much money will she have in 3 days, 4 days, 5 days? Can you find out how much money she will have in 7 days without filling out the entire table?

Figure 6.16 Table Problem

Day	Operation	Money
1	2 × 1 + 10	$12
2	2 × 2 + 10	$14
3	3 × 2 + 10	?
4		$18
5	5	$20

Another Example

Mr. Michael is a carpenter. He has several big pieces of wood that he bought by the yard. He now wants to cut it into bookshelves. Each shelf is 1 foot long. He has 1 piece that is 1 yard. He has 3 pieces that are 2 yards. He has 5 pieces that are 5 yards. How many shelves can he cut altogether?

Figure 6.17 Table Problem

Yards of Wood	# of Shelves
1	**3**
2	
3	

The How Many Animals and Legs Problems?

These are common problems. There are many different types of these problems with either two or three or sometimes more things to think about (see Figures 6.18, 6.19 and 6.20). They develop logical thinking and reasoning. They are marvelous problems that can be ramped up as understanding is built.

Figure 6.18 Table Problem

Version 1

Dogs	Legs
1	**4**
2	**8**
3	**12**
?	**16**

How Many Legs?
A dog has four legs. 
How many legs does 2 dogs have? 
How many legs do 3 dogs have? 
If there are 16 legs, how many dogs are there?

Figure 6.19 Visualizing the Problem

Version 2

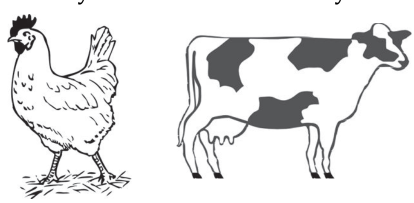
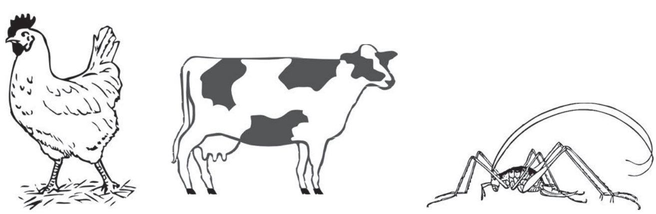
In the farmyard there are 6 legs. There are chickens and cows. How many chickens and how many cows are there? 
In the farmyard. There are 8 legs. There are chickens and cows. How many chickens and how many cows are there?
In the farmyard there are 10 legs. There are chickens and cows. How many chickens and how many cows?
Make up your own chickens and cows problem.

Figure 6.20 Visualizing the Problem

Version 2

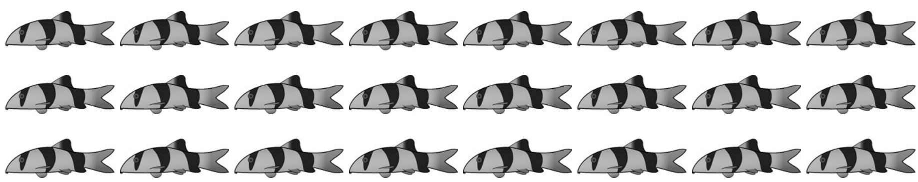
In the farmyard there are 24 legs. There are crickets, chickens and cows. How many crickets, chickens and cows are there? 
In the farmyard. There are 30 legs. There are chickens, cows and crickets. How many of each animal could there be if there were some of all the animals?
Make up your own chickens, crickets and cows problem below:

Reasoning About Remainders

Remainders are difficult. But they don't have to be. Start out by teaching remainder problems with pictures. Students should be able to reason about the numbers by seeing them. They should talk about various situations to understand that we either round up, round down, drop or leave the remainder (see Figures 6.21 and 6.22).

Figure 6.21 Act Out, Sketch or Diagram the Problem

There were 24 fish in the tank at the pet store. The clerk took them out and put them in smaller bowls with 5 fish in each bowl. How many bowls did he use?



Graphic organizers can help students to think through the process.

Figure 6.22 Act Out, Sketch or Diagram the Problem

Figuring out remainders: **Pick a problem** **Record it. Solve it. Write the Remainder. Interpret it!**			
Problems	**Model**	**Remainder**	**Round Up/Down/ Drop/Leave**
Version 1: The 3rd graders are going on a field trip. There are 67 students and 5 adults. Each minivan fits 15 people. How many minivans do they need?	72/15 = So we need 5 buses because 4 would only fit 60 people and 12 would be left over.	12	In this case, we have to round up. We need 5 buses because the 12 people need a place to sit.

Version 2:

The third-graders are going on a field trip. They ordered 5 vans. If each van can seat 15 people, did they order enough vans so that everyone has a ride?

Yes or No?

Explain your thinking.

Other Type of Remainder Questions

Look at these different remainder problems and think about the different levels of cognitive demand of each of the problems. Some are straightforward, while others involve multiple steps. It is important to expose your students to a variety of remainder problems that ask them to do different things so that they become flexible thinkers.

Grandma made 72 cupcakes for the birthday party. She divided them equally into 10 boxes. Does she have any *left over*?

Adapted from NAEP 1996 Grade 8

At the birthday party, Tim's mom made 30 hotdogs for 17 children. If each child is to have at least one hotdog, how many children can have more than one?

Adapted from National Assessment of Educational Progress, 1992, Grade 4 and Grade 8 Mathematics Assessments

The bakery must buy sugar flowers to go on top of the cupcakes. There are 12 cupcakes in a box. They need to decorate 15 filled boxes. Sugar flowers are sold in packages of 30. How many packages of 30 does the bakery need to buy to have enough sugar flowers?

Luke and Mattie bought a video game. They each saved $26.85. The game cost $50. If they split the left over money evenly, how much did they each get back?

Baseball cards were on sale at the toy store. They cost $1.50 each. Nathan had $33. How many can he buy? If his mom will give him enough to buy one more, how much will he need from her?

Students should have many opportunities to work with decimal problems.

My dad has 24 feet of wood. He is going to make 5 bookshelves. How much wood would he use for each shelf if they are going to be the same size?

Students should know that in this type of measurement problem, we can keep the fraction remainder as part of the answer.

Thinking About Others Thinking

There is a specific genre of word problem types that ask students to think about the thinking of others and then respond (see Figure 6.23). Here are a few of the problems in this genre:

- Can you use Mike's strategy?
- Can you convince me?
- Can you find and fix the error?
- Two-argument problems

Can You Think Like Mike?

Figure 6.23 Unpacking Strategies

Mike said that when he saw 125 ÷ 25, he instantly thought, "What times 25 equals 125?" Can you use his strategy to think about 400 ÷ 8?

A. Solve using Mike's method.

Step 1: $8 \times ? = 400$
Step 2: We know that $8 \times 5 = 40$
Step 3: So $8 \times 50 = 400$

B. Explain what you did.

I know that $8 \times 5 = 40$; therefore, ***40*** ÷ 8 = ***5*** so we now understand that ***40***0 ÷ 8 = ***5***0

Convince Me Problems

Convince me problems are set up so that students can actually prove their thinking in a logical format. These problems require students to use numbers, words and pictures to explain their thinking. They demand that students justify what they know and how they know it. They require students to lay out that justification in an organized manner. Here is an example (see Figure 6.24):

Figure 6.24 Use Manipulative, Sketches or Diagrams

Dan said that $\frac{1}{4}$ was greater than $\frac{1}{5}$. Discuss his thinking.

(Convince me with words.)

I know that. ____
____
____
____

(Convince me with drawings or a diagram.)

I can prove my thinking with a model.

(Convince me with numbers.)

I can verify the answer another way:

Therefore,

Adapted from http://mason.gmu.edu/~jsuh4/teaching/convince.htm

Find and Fix the Error

In this activity you give the students a problem with an error. The students have to find and fix the error. This develops what research calls "a nose for quality." Students are able to think about, analyze and reason to themselves and with others about the nature of the solution. They ponder, is it correct? They think about where the error might be and then how they might fix it. This can occur across mathematical topics and should because there are specific errors that students consistently make (like adding denominators). The find and fix the error activity spotlights these common errors and asks students to *catch them and throw them back fixed. For example:*

Luke read the following problem: *Grandma Betsy made some brownies. On Monday her grandchildren ate $\frac{1}{3}$ of them. On Tuesday they ate another $\frac{1}{3}$ of them. How much did they eat altogether both days? Luke said $\frac{2}{6}$ of the brownies. Kelly said that didn't make sense. Who is correct and why?*

Linda answered the following problem like this:

$$\begin{array}{r} 2500 \\ -\ 498 \\ \hline 2{,}198 \end{array}$$

Can you find and fix her error?

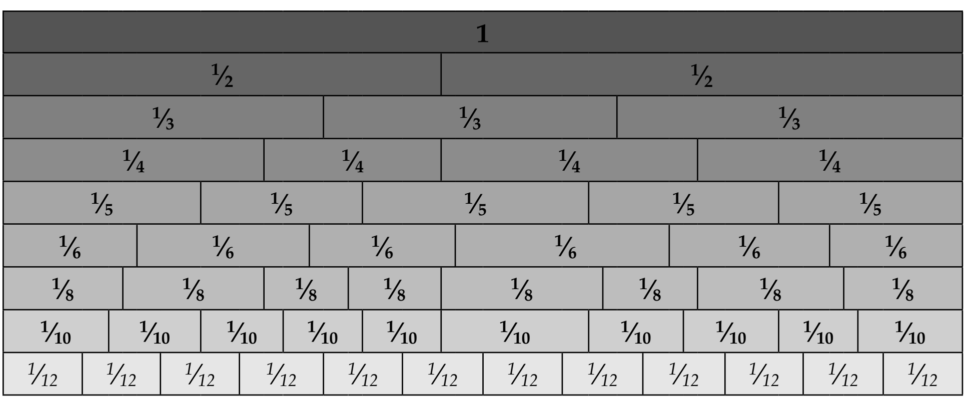
*Mary multiplied 478 × 20. She got 5,039. Use estimation to explain why this answer is not reasonable.

Two Argument Problems

Two argument problems are great! They get students to think and reason to themselves and out loud. The makeup of these problems is to have a central problem with students, approaching it from different perspectives. The students have to read the problem and then make sense of it and decide who is correct. For example (see Figures 6.25 and 6.27).

Mary Jo said that $\frac{3}{8}$ is bigger than $\frac{2}{4}$ because 8 is bigger than 4. Kylie said that you can't just say that. You must look at the size of the fraction. Who is correct? Use your tools to explain your thinking.

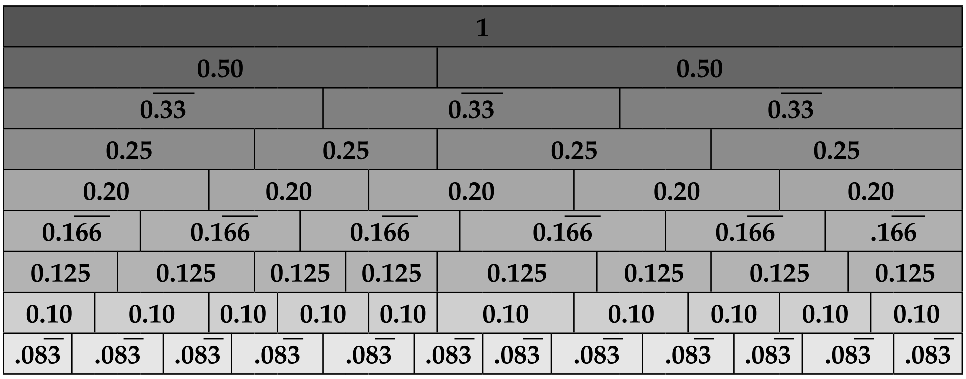
Figure 6.25 Fraction Bars



Decimal bars are just as helpful as fraction bars. Here is an example (see Figure 6.26).

Ken said that 7 hundredths is more than 2 tenths because 7 is greater than 10. Michelle disagreed. She said that 2 tenths is bigger because tenths are bigger than hundredths. Who is correct? Use a model to justify your thinking.

Figure 6.26 Decimal Bars



Using templates for two argument problems is also a way to scaffold thinking.

Figure 6.27 Use Manipulatives, Sketches or Diagrams

Problem: Todd said that he ate ¼ of his candy bar. Mary said she ate ½ of hers. Mary said she ate more than Todd because ½ is always bigger than ¼. Todd disagreed.	
Think about the arguments. ***Who do think is correct?***	**Decide and defend your thinking.** ***Why did you think this?***
Prove your thinking with a model.	**Prove your thinking with numbers.**

Another genre of reasoning problems is where students have to pick which statement is true given a variety of options (see Figures 6.28 and 6.29). This problem is centered around comprehension and language.

Figure 6.28 Reasoning About Numbers

Mike and Mary are having a math discussion. They are talking about which number will make this equation true. Mike says that the answer would be 56. Mary says the answer would be 8.

$64 = b \times 8$

Part A: Who is correct?

Answer: ________________________

Part B: Explain your thinking with numbers, words and pictures.

Figure 6.29 Reasoning About Numbers

A busy candy store in downtown New York City sells about 25 lbs. of chocolate a day. During the month of February, they sell three times as many pounds of chocolate a day. Which of the following is true?

A. Every week (7 days), they sell over 200 lbs. of chocolate.
B. In February, they sell 1,000 pounds a week.
C. In February, they sell about 500 pounds a week.
D. Every week (7 days), they sell 125 lbs. of chocolate.

Adapted from NAEP, 2011

Reasoning about numbers: greatest differences or smallest sum (etc. . . .)

These problems are popular abroad in many top-performing math countries (see Figures 6.30 and 6.31).

Figure 6.30 Example 1

Tomas played a game with his friend. In this game they pick cards and then have to make the largest sum. Carlos picked 2,9, 7 and 4. How can he arrange these numbers to get the largest sum?

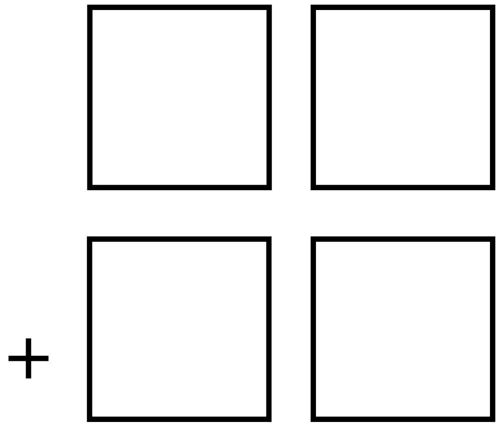
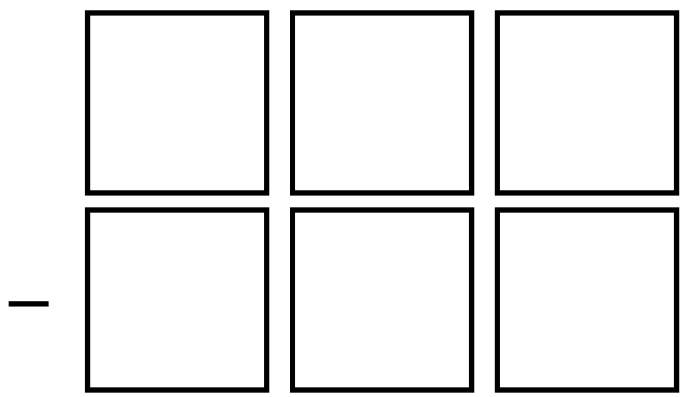


Figure 6.31 Example 2

Tomas played a game with his friend. In this game they pick cards and then have to make the largest difference. Carlos picked 1, 9, 7, 5, 0 and 4. How can he arrange these numbers to get the largest difference?



Explain your thinking.

Key Points

- Students should have many opportunities to do pair and group reasoning activities.
- Students should contextualize problems weekly.
- We must focus on both set-up and solution equations.
- Graphic organizers help scaffold reasoning.
- The 2- and 3-bean salad problems help to practice algebraic thinking.
- Coin puzzles help to practice algebraic thinking.
- Concentration matches get students to focus.
- Venn diagrams help students to reason.
- Table problems help students to see patterns.
- Leg problems help students to reason and see patterns.
- Students should be able to analyze different types of remainder problems.

Summary

Reasoning problems should be an essential part of your curriculum. Of course, students are supposed to reason through all types of word problems, yet the problems in this chapter offer specific types of problems to organize thinking. Students should practice problem types like these so that they gain the necessary skills in thinking about problems, organizing information, discussing them and reflecting on their process. Matrix problems, Venn diagrams and tables all require that students stop, think logically and go step by step. As students do this, they become stronger, more adept thinkers. They learn that they can work through problems that may at first seem to give a lot of information, but that different types of organizers help to navigate that information. We need our students to develop problem-solving skills and to have a repertoire of tools to organize their thinking. Reasoning problems help students to develop that mental muscle.

Reflection Questions

1. In what ways do you currently perform reasoning problems with your students?
2. How often do you ask your students to make up their own word problems? How do you scaffold this process so that they are successful?

3. Do you do a variety of types of reasoning problems, including Venn diagrams, table problems, leg problems, greatest difference and smallest sum problems?
4. What are three big takeaways from this chapter? Where will you start?

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7

Modeling Thinking

> *When we are teaching strategies for problem solving, we must be facilitative rather than prescriptive.*
>
> *(Willis and Fuson, 1998)*

It is important that students act out problems, model them with concrete materials, draw them and make diagrams. In the upper elementary grades, we rarely act out problems. We can and need to do better. Any unit of study can be approached from a hands-on, connected to real life, engaging manner. Pape (2004) notes that

> If students are encouraged to understand and meaningfully represent mathematical word problems rather than directly translate the elements of the problems into corresponding mathematical operations, they may more successfully solve these problems and better comprehend the mathematical concepts embedded within them.

For example, if we are learning about division, we should be acting out the problems in front of the class. If we are learning about liquid volume, then we should be making punch or measuring milk in containers. We need to make sure that students understand the connections to real life. All students should have toolkits to help them model their thinking.

Math Toolkits

A math toolkit has two parts. It has the manipulatives and the templates. For instance, in a unit on fractions, the toolkit might have fraction circles, squares and strips. The templates could be laminated versions of these things. The templates would also include fraction number lines. When combined, these power tools help to scaffold even the hardest of word problems and make them all doable! In addition to the physical tools, students should also work with virtual tools (see Figure 7.1). For example, the students might work with the physical base ten blocks one day and then the next time, they might work with base ten block paper. In another lesson, the students might work with base ten sketches and then on another day they might work with the virtual base ten blocks. There are many different ways to access these virtual blocks. One site that I like in particular is the NLVM (the National Library of Virtual Manipulatives).

Figure 7.1

Five great websites for virtual manipulatives are:

- Math Playground Math Bars
- NLVM
- Glencoe
- NCTM Patch tool
- Math Learning Center Apps

Acting Out and Concrete Materials

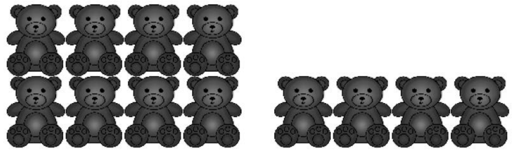
Students should be acting out problems with a variety of concrete manipulatives. Remember that there are different levels of manipulatives. Some are more abstract than others. For example, if we are telling division problems, and we have little plastic people or little paper dolls, this is different from using cubes to represent the people.

Manipulatives are important when it comes to problem solving. Van de Walle says that they are "thinking tools" (2007). He cautions us though to not let the tools become the center of the math. The math should be the focus of the study. The tools should not overshadow the math or overscaffold the math. That said, there are some great manipulatives that are essential for problem solving in the upper elementary grades.

A Closer Look at Math Manipulatives: Bears

Bears are a great tool (see Figure 7.2). They are underutilized in the upper elementary grades. We should be using them through middle school. In the primary grades, use them to solve the basic CGI problems. In the upper

Figure 7.2



http://www.KinderAlphabet.com

elementary grades, use them for multiplication, division and comparison problems as well as for fraction problems.

John had 4 bears. Mary had 2 times as many as he did. How many how did she have? How many did they have altogether?

There were 8 bears. Some more came and now there are 16. How many came?

There were 15 bears equally divided into 5 boats. How many bears were in each boat?

There were 12 bears. One-third of them were blue. One-half of them were orange. The rest were yellow. How many of each bear were there?

There were 15 green bears. This was 5 times as many as the orange bears. How many orange bears were there?

Unifix Cubes

Unifix® cubes should be used for the same types of problems as the bear problems. Also use Unifix® cubes and snap cubes for perimeter and area problems (see Figures 7.3, 7.4 and 7.5). Use 1-inch tile paper to act out these problems as well. This paper allows students to trace their work on the grid paper.

Farmer Betsy-Lou planted a garden. It had an area of 12 feet. What are the different shapes that this garden could have been?

Figure 7.3 Chelsea Shoeck, cschoeck@live.com

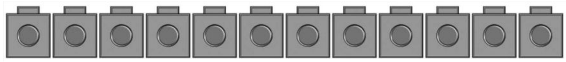


Figure 7.4

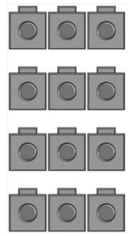
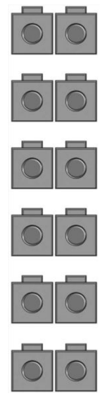


Figure 7.5



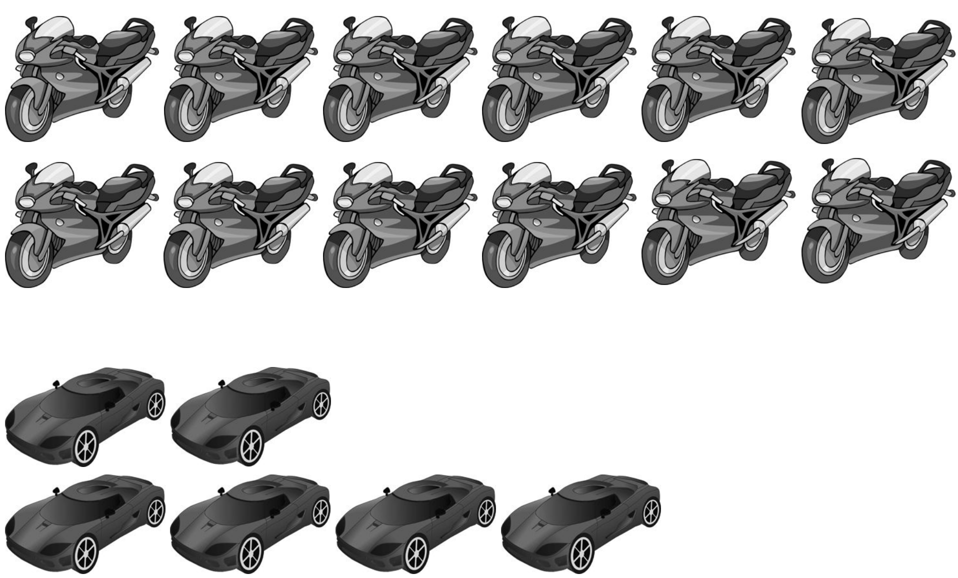
Farmer Betsy-Lou planted a rectangular garden. The garden had an area of 20 feet. One side was 5 feet. What was the other side?

Small Toys of Real Things (fruit, animals, vehicles)

The use of actual small toys that represent real things is a great motivator and interest grabber (see Figure 7.6).

Joe had several toy motorcycles. He got 5 times as many as he had and now he has 12. He has half as many cars as motorcycles. How many toy vehicles does he have altogether?

Figure 7.6



Cuisenaire® Rods (bring them back . . . search the basements and closets)

Cuisenaire® rods[1] are the forgotten manipulative. They are great to teach the basic operations as well as fractions. Every school over 25 years old has them hidden somewhere deep in a closet because they used to be as ubiquitous as the teddy bears are today. Here is an example (see Figure 7.7).

Figure 7.7

Mr. Tom was cutting wood to make some shelves. He had a piece of wood that was 10 feet long. He wanted to make shelves that were 2 feet long each. How many shelves can he make out of that 10-foot piece of wood?

1. Model this problem with the Cuisenaire® rods.

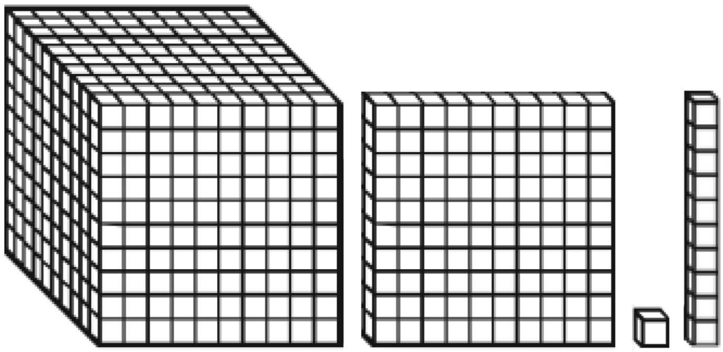
2. Sketch a picture of your model.

3. Write an equation to solve the problem.

Place value blocks

Place value blocks should be used frequently so that students understand the structure of numbers (see Figure 7.8). Have students solve different types of problems with them. Remember the original theory said that students should generate a number, then build the model, draw a picture of the model and then do the calculations so that they could see the complete cycle. So for example, you might have a problem like: *Grandma planted a garden that was 11 feet by 12 feet. She is going to divide it into 4*

Figure 7.8



different parts (see Figures 7.9 and 7.10). The corn will have the largest part. The green beans will be planted in the second largest area. The carrots will cover the third-largest area and the potatoes will be in the smallest area. Show what her garden will look like? What is the total area of her garden? Have the students sketch out the place value block representation and explain it.

Figure 7.9

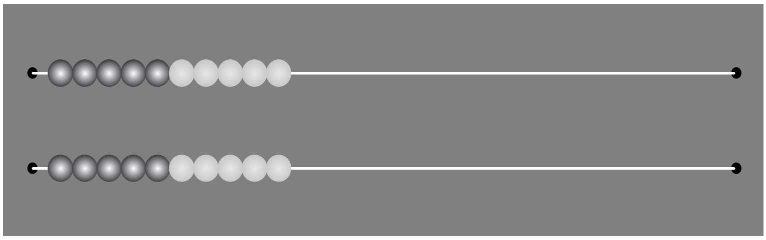
	10 +	2
10 +	Corn 10 × 10 100 sq. ft.	Green Beans 10 × 2 20 sq. ft.
1	Carrots 1 × 10 = 10 sq. ft.	1 × 2 = 2 sq. ft. potatoes

Figure 7.10

	10 +	2
+ 10	10 × 10 = 100 sq. ft.	10 × 2 = 20 sq. ft.
1	1 × 10 = 10 sq. ft.	1 × 2 = 2 sq. ft.

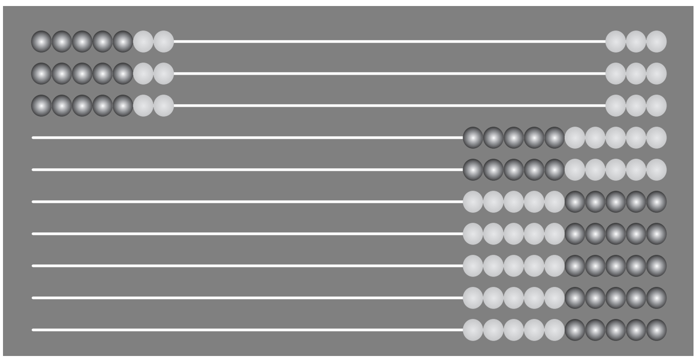
The Rekenrek

Figure 7.11 Rekenrek (Number rack)



http://www.mathlearningcenter.org/web-apps/number-rack/

Figure 7.12 Virtual Rekenrek (number rack)



http://www.mathlearningcenter.org/web-apps/number-rack/

The Rekenrek is a great tool for teaching a variety of strategies (see Figures 7.11, 7.12, 7.13, 7.14 and 7.15). The Rekenrek (or counting rack) is often neglected in the older grades, but it is a great tool to use for both multiplication and division as well as multiplicative comparison problems. When I am trying to introduce the three different types of multiplicative comparison problems, I use the Rekenrek. It allows me to use small numbers so we can concentrate on the concept (later after the students understand the problem, we can use larger numbers and we will have phased out this scaffold).

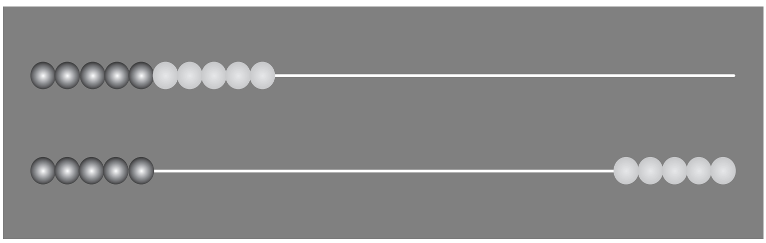
Problem A: *Sue has 2 marbles. Her sister has 2 times as many as she does. How many does her sister have? How many do they have altogether?*

Figure 7.13

http://www.mathlearningcenter.org/web-apps/number-rack/

Problem B: *Sue has 10 marbles. She has 2 times as many as her sister. How many does her sister have? How many do they have altogether?*

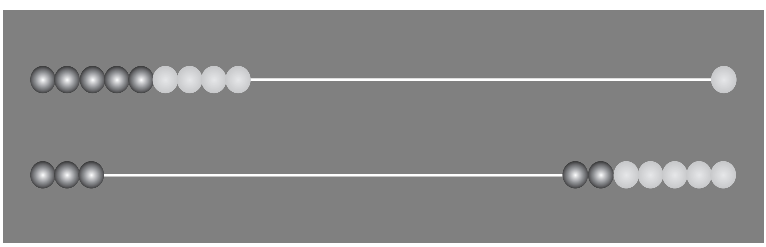
Figure 7.14



http://www.mathlearningcenter.org/web-apps/number-rack/

Problem C: *Sue has 9 marbles. Her sister has 3 marbles. How many times as many marbles does Sue have as her sister? How many do they have altogether?*

Figure 7.15



http://www.mathlearningcenter.org/web-apps/number-rack/

Problem D: Expert Extension to above problem: *If her sister got some more marbles and now she has 2 times as many marbles as Sue, how many did she get?*

Tape Measures, Yard Sticks, Meter Sticks and Rulers

If you are going to have the students solve length problems, then use tools that measure length (see Figure 7.16). Use tape measures, rulers, yard and meter sticks. It is important that students can experience measurement problems rather than just imagine them. For example: *Mrs. Clay had a cotton-ball throwing contest for the Math Olympics. Tom tossed the cotton ball 3 times. First he tossed it 12 inches. Then he tossed it 19 inches and then he tossed it 10 inches. Were his total tosses more than a yard? Use your measurement tools to calculate Tom's tosses.*

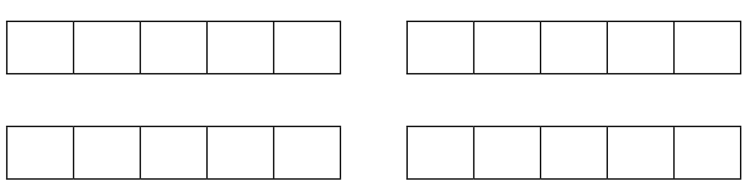
Figure 7.16

Templates (Should be Laminated or Placed in Sheet Protectors)

Number Frames

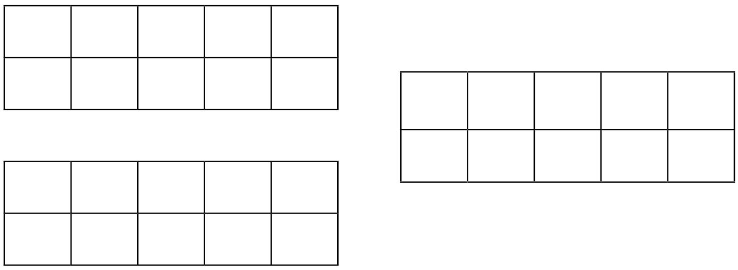
I want to talk about the use of number frames for teaching visual multiplication and division. Students use these frames a great deal in the primary grades. When they are learning to multiply and act out problems, 5 frames are great for 5s and 10 frames are great for 10s. For example, *Ted had 4 boxes with 5 marbles in each box. Use the 5 frame templates to illustrate your problems* (see Figure 7.17).

Figure 7.17



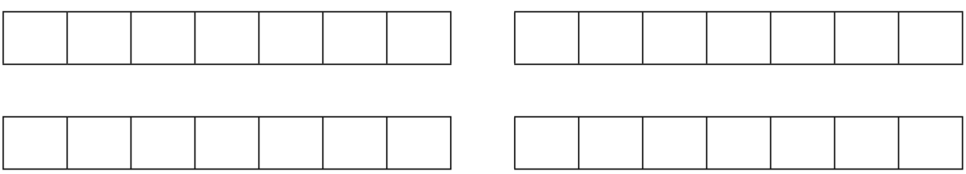
The second frame to introduce is the 10 frame. The 10 frames are great for teaching students the patterns of dividing 10s. For example, *Sue had 30 marbles divided into groups of 10. How many groups did she have? Use the 10 frames to solve* (see Figure 7.18).

Figure 7.18



Another frame that I like is the 7 frame. Sevens are notoriously difficult for students. So, use weekly pill boxes to have students solve problems from the 7 tables. For example, *The bakery had boxed its cookies in groups of 7. If they had 21 cookies, then how many boxes did they need? Use the 7 frames to model your thinking* (see Figure 7.19).

Figure 7.19



Number Lines, Number Ladders and Open Number Lines

Number Lines

Students should work often with number lines (see Figures 7.20, 7.21, 7.22, 7.23 and 7.28). They should have all types with various number ranges. They should have some premade and some that they make. By second grade, students are expected to add sums within 100. This can be very tricky to teach. Using number lines and number grids as scaffolds can help tremendously.

With number lines, you want students to begin to compose and decompose numbers. You start out with easy problems, and you scaffold up to more difficult ones. So a starter problem might be: *Susie had 10 lemons. Her friend gave her 10 more. How many does she have altogether now?* In this type of problem, the children start jumping 10s quite comfortably. They do this on the number grid as well, where they simply have to slide down.

After the children become comfortable skipping 10s, you introduce some 5s. You might say: *Susie had 15 lemons and her brother gave her 25 more.* The children would then start at the large number and break the smaller number up into 10s and 5s. So, they would jump 25 to 35 and then 5 more. Once the children are really comfortable jumping around between 10s and 5s, then you introduce some numbers where they have to decompose into 1s.

Figure 7.20

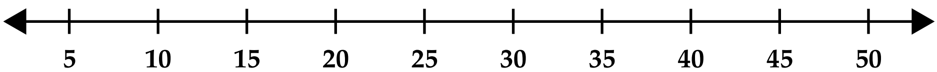


Figure 7.21

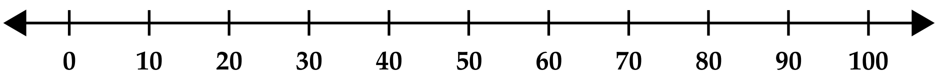
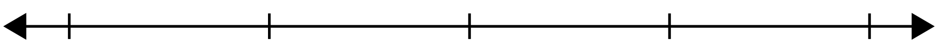
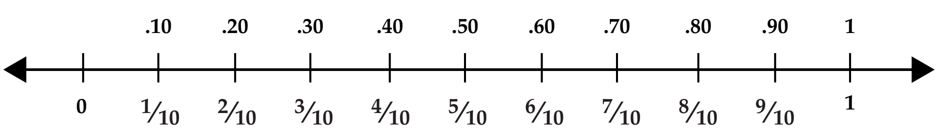


Figure 7.22



Don't just stop with whole numbers. Make sure that students have fraction and decimal number lines, depending on the content that they are studying. Have students engage in a variety of routines throughout the week where they have to reason on the number line.

Figure 7.23



The Open Number line

After the students have had plenty of experiences with the number grid and the number line, you want them to move to the open number line. The open number line is a power tool—one that promotes powerful mathematical thinking. It helps children to show and explain their invented strategies, builds flexibility with numbers and scaffolds the mental representation of number and number operations to support mental arithmetic strategies (Beishuizen, 1999; Fosnot & Uittenbogaard, 2007).

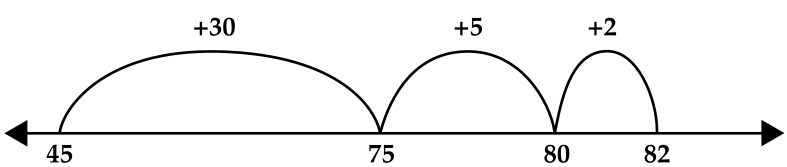
The open number line gives students a model for representing their thinking. It requires that they be actively engaged in their explanations. It is more cognitively demanding than either base 10 blocks or the 100 chart according to Klein, Beishuizen and Treffers, 2002 cited in Fosnot, 2007. It is great to use this strategy with the whole group, but if you really want to have in-depth conversations, then you should do it in small guided math groups so children can have the time to explain their thinking. In a guided math group, the teacher would model the use of the open number line and then give the students the opportunity to work on a problem together with the open number line. Finally, the teacher would give each student an opportunity to solve a problem using the open number line as the model while explaining their thinking to the group.

Look at this problem: *Johnny had 57 marbles. His brother gave him 26 more. How many does he have altogether now?* On the open number line, the child might jump from 57 and then break the 26 into two 10s and go 67 and then 77 and then go to the nearest 10, which would be 80 plus 3 more to complete the 6, which would be 83.

57 67 77 80 83

To use the open number line, students draw a line, plot numbers on it and count using a variety of strategies. For example, let's take the problem 45 + 37 (see Figure 7.24).

Figure 7.24



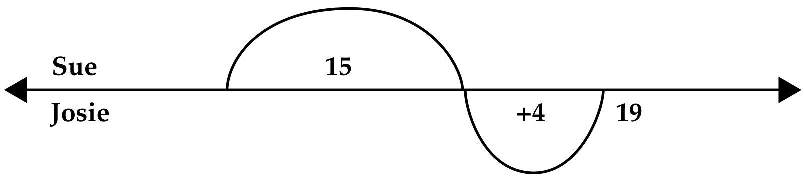
The student starts at 45 and jumps 30 because they broke apart the 37 into 30 and 5 and 2. From 75, they jump 5 more to 80 and then 2 more to 82. Open number lines are a huge part of every state's math standards, and it is important to make sure that students are comfortable using them.

Double Open Number Line

The double number line is a great model for comparing two different things. For example, *Sue had 15 apples and Josie had 4 more than she did. How many did Josie have?*

Students draw a line and then plot one part of the comparison on the top and the other part of the comparison on the bottom (see Figure 7.25).

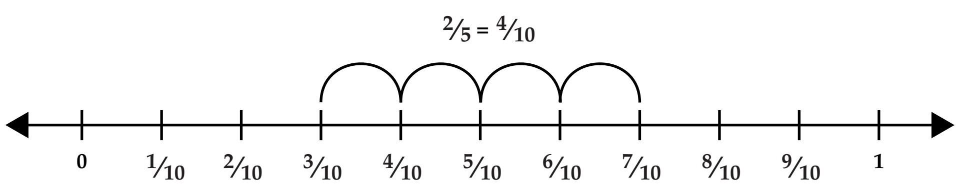
Figure 7.25



Double Number Line for Fraction Problems

Sue walked 3⁄10 of a mile in the morning. In the afternoon she walked 2⁄5 of a mile. How far did she walk altogether? Using a double number line students would first think of a common denominator. Then they would plot one of the numbers. In this case, we can use tenths as our common denominator and plot 3⁄10. We know that 2⁄5 is equivalent to 4⁄10 so we just hop on. So we know that 3⁄10 + 4⁄10 = 7⁄10 (see Figure 7.26).

Figure 7.26



Let's look at another problem. Let's say *Joe ran 2⁄6 of a mile in the morning and 2⁄3 of a mile in the afternoon. How far did he run altogether?* We know that 2⁄3 is equivalent to 4⁄6 so we just count on. We get 6⁄6, or 1 whole mile (see Figure 7.27).

Figure 7.27

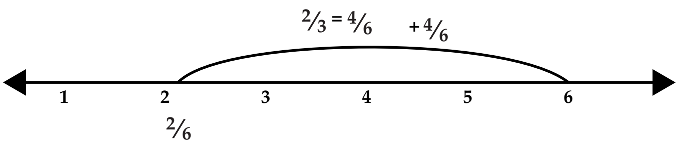


Figure 7.28

> **Virtual Number Lines:**
>
> - Math Learning Center Apps
> - NLVM
> - Fuel the Brain
> - Dream Box

The Number Grid

The number grid is an excellent tool to help students develop efficient mental strategies. Students can initially use the number grid as a scaffold to break apart numbers, jump to friendly numbers and add or subtract quickly. For example, on the number grid to *add to result unknown problems*, students choose a number to start with and then add. You want to teach them how to break apart numbers and use this method to add efficiently. Let's look at this on a number grid. For example, you might say, *Olga had 45 apples and her mother gave her 27 more.* How could we hop up the number grid easily? Let's think about getting to some friendly numbers. We always try to find a 10. So we would jump 45, 55, 65 . . . that gives us the 20. Then, we would break up the 7 into 5 and 2 . . . so 70 and then 72 (see Figure 7.29).

Figure 7.29

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

In the upper elementary grades, you should use the decimal number grid when solving word problems (see Figure 7.30).

> For example, Claire decided to save her money. She saved a penny the first day. She doubled that the next day. She then doubled that amount the next day. How much money will she have saved on the 5th day? How many days will it take her to save a dollar?

Figure 7.30

0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1
0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.2
0.21	0.22	0.23	0.24	0.25	0.26	0.27	0.28	0.29	0.3
0.31	0.32	0.33	0.34	0.35	0.36	0.37	0.38	0.39	0.4
0.41	0.42	0.43	0.44	0.45	0.46	0.47	0.48	0.49	0.5
0.51	0.52	0.53	0.54	0.55	0.56	0.57	0.58	0.59	0.6
0.61	0.62	0.63	0.64	0.65	0.66	0.67	0.68	0.69	0.7
0.71	0.72	0.73	0.74	0.75	0.76	0.77	0.78	0.79	0.8
0.81	0.82	0.83	0.84	0.85	0.86	0.87	0.88	0.89	0.9
0.91	0.92	0.93	0.94	0.95	0.96	0.97	0.98	0.99	1

Math Mats

Math mats are great ways to get students to problem solve (see Figure 7.31). They provide context by giving a picture of some type of background (a circus, a playground, etc.). This context is a scaffold that helps students understand what they are doing. Math mats are a type of graphical organizer that allows students to see and act out the problems they are trying to solve.

Math mats also help to relieve some of the anxiety that word problems cause among students. Kouba, Brown, Carpenter, Lindquist, Silver & Swafford (1988) found that word problems cause a great deal of anxiety, which I would say hinders students' problem-solving abilities, often even before they get started. Math mats can help to ease some of that anxiety by tapping into the familiar. You can use math mats to introduce, demonstrate, reinforce, reteach and differentiate story problem types. They also provide students with a springboard for telling their own stories.

Figure 7.31

Great Math Mat Resources:

Sparklebox (look under calculations and the specific operation)

http://www.kidsparkz.com/theme-a-pedia.html

https://www.pinterest.com/drnicki7/math-mats/

To do any of this though, you have to start with a great math mat. When using math story mats, it is important to have a focus in the guided math group. Think about what types of stories you are focusing on in that particular session. CGI provides excellent resources for understanding and teaching story types. You want to make sure that students have some concrete experiences, some pictorial experiences and some abstract experiences.

With math story mats, I would start by acting out the stories with manipulatives. Later, I would have the students draw out the stories. Finally, I would have them write the words and symbols connected to the stories. For this step, I would have number cards and math symbols so the students could show the number models. I would also have them write out the number models. Sometimes I would have them match the story problem with the number model and then model it on the math mat.

Grouws and Cebulla (2000) found that working with partners for problem solving increases student achievement. They also found that long-term use of manipulatives increases math achievement. With story mats, you can have students working with partners and using manipulatives to represent their thinking.

- Have stories written on task cards. The students pull the cards and model the story with manipulatives to solve it.
- Have students work in partners and one partner tells a story while the other one represents it on the math mat.
- Have students tell their own story and represent it on the math mat as they are telling it and others listen.
- Have a student tell a story and the group represents and solves it on their own individual math mats.
- Remember to always have a small group discussion after the practice so students can concretize their learning. Grouws and Cebulla (2000) found that this discussion helps to raise student achievement because students get to hear others and think about their own reasoning. Math mats are an abstract scaffold. They provide a context but are much more open than the other tools previously mentioned.

Drawings and Sketches

After the students have worked extensively with the concrete materials, have them draw what they are doing. It is important to teach the students to do "mathematical sketches," rather than to have them spend hours drawing a marble.

(Bar/Tape/Strip) Diagrams

After the students have practiced using pictures and drawings, they should be doing diagrams. Bar/strip/tape diagrams are important in terms of unpacking word problems. Most of the time, once students have set up the strip diagram, the calculations are easy because they actually understand what they are doing.

Charles states that bar diagrams are a visual approach to teaching word problems. He notes that

> problem solving is grounded in reasoning. Quantitative reasoning involves identifying the quantities in a problem and using reasoning to identify the relationship between them.
>
> (http://assets.pearsonschool.com/asset_mgr/current/201218/MatMon110890Charles_SWP_Revise_eBook.pdf p.5)

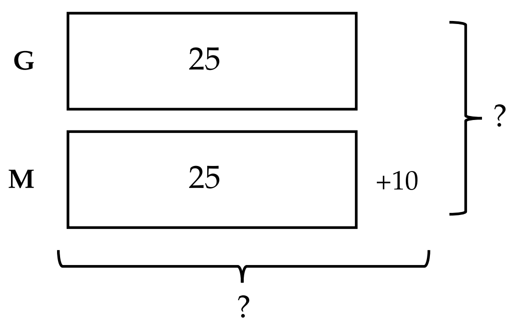
For example,

John has 25 green marbles in his collection. He has 10 more multicolored marbles than green ones. How many multicolored marbles does John have? How many does he have altogether? (See Figure 7.32.)

The quantities in this word problem are:

- The number of green marbles (a known value 25)
- The number of multicolored marbles (an unknown value)

Figure 7.32



As Diezmann and English note, "a diagram can serve to 'unpack' the structure of a problem and lay the foundation for its solution" (cited in Charles, Monograph 24324). Nickerson found that the ability to use diagrams is integral to mathematics thinking and learning (cited in Charles). Other researchers found that

> training children in the process of using diagrams to [meaningfully represent and] solve [mathematical word] problems results in more improved problem-solving performance than training students in any other strategy.
>
> (Yancey, Thompson & Yancey, cited in Charles)

Charles emphasizes that the most important thing is to get our students thinking about the relationships between these quantities. Often, students rush to the operation without fully understanding the problem. They are often taught to solve the problem without even representing it or really understanding it. Charles states that

> the challenge is to identify statements in the problem that express relationships between quantities, to understand those relationships, and to choose an appropriate operation or operations to show those relationships.
>
> (p. 5)

In the problem about John's marbles, the relationships are:

There are 25 green marbles.
There are 10 more multicolored ones than green ones.

Look at how this bar diagram of the word problem about John's marbles represents the quantities and their relationships.

Green marbles
Multicolored marbles

The diagram clearly illustrates what we are talking about when we say "10 more than the green ones." It makes the math visual. It brings the problem into 3-D. The relationships among the quantities is clearly, visually apparent because of the bar diagram.

- There are 10 more multicolored marbles than green ones.
- The 2 boxes that contain 25 marbles and then the one that shows 10 more shows this relationship.

So now that we have established the relationships, we just have to do the calculations. We have to know how to translate these relationships into numerical expressions. We now have to rely on our understanding of operations. So we can add the 25 plus 10 more. We can easily see where we got this from by looking at the bar diagrams. So, the answer is John had a total of 35 multicolored marbles altogether.

Bar diagrams help students to see the problem and give a 3-D view of the relationships. They also help students to lay out the situation. They encourage students to think before they try to solve. Teachers can introduce bar diagrams as early as kindergarten.

When students understand how to make different diagrams, they can use this strategy for solving multi-digit problems. So we must start out with easy problems and single-digit numbers and scaffold up to multi-digit numbers.

Key Points

There are many different tools that students should use to solve word problems including:

- Math toolkits
- Manipulatives:

  bears
  Unifix® cubes
  Cuisenaire® rods
  place value blocks
  Rekenrek
  tape measure

Templates

- number frames: 5, 10, 20
- 100s grids
- decimal hundredths grids
- 10,000 grid (Van De Walle)
- number lines
- double number lines
- number ladders
- math mats

Drawings/Diagrams

- mathematical sketches
- bar/tape/strip diagrams

Summary

Math toolkits are essential for helping to scaffold students' mathematical experiences. They should include concrete manipulatives, templates and tools. There should be a variety of concrete manipulatives including bears, Unifix® cubes, Cuisenaire® rods, place value blocks, Rekenreks® and tape measures. There should also be a variety of templates such as number frames, various types of number grids, various types of number lines and ladders and storytelling mats. Students should also use drawings and diagrams to show their thinking. It is crucial that students are given plenty of opportunities to explore and use a variety of tools, templates and drawings to model their thinking.

Reflection Questions

1. Do you presently have toolkits in your classroom? Does every student have his/her own set of manipulatives?
2. Do your toolkits include both templates and manipulatives?
3. Does your program focus on students modeling their thinking?
4. What are your biggest takeaways from this chapter?

Note

1. Cuisenaire® Rods are a registered trademark of the The Cuisenaire® Company.

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8

Mini-Lessons

Springboards into Great Word Problem Premises

Math Mentor Texts, Poems, Songs, Believe It or Not Stories and Shared Experiences

> *Word problems don't have to be boring! We must bring the imagination of Disney® to the situation. If done right, students can be taught to love word problems like they love good movies.*

There are many different ways to get students interested in story problems. Teachers should use math mentor texts, *believe it or not stories*, real-life stories, poems and shared experiences. All of these types of story starters can be informative, instructive and engaging. They also serve dual purposes, because you can use them for not only math but also other areas in the curriculum such as science, social studies and even art. There are so many resources available to get you started down this exciting path. In this chapter, I will discuss some of those resources.

Math Mentor Texts

Math mentor texts are picture books that are written with a math premise. There are so many good books out there now and many of them have been recorded into videos. Use these as springboards into understanding the math through story problems. I have created tables of a few of my favorites (see Figures 8.1, 8.2, 8.3 and 8.4). What I truly love about many math mentor texts is that there are tons of math lesson plans that go along with them to help teach the concepts. Furthermore, many have video versions as well. Let's take a look at a few lessons.

Figure 8.1 Great Math Picture Books

Math Text	***The Lion's Share*—Matthew McElligott**	***My Full Moon Is a Square*—Elinor J. Pinczes**	***Actual Size*—Steve Jenkins**	***A Fly on the Ceiling*—Julie Glass**
Storyline	This story is about a dinner that takes place at the Lion King's home. The animals come and eat and then share a cake. They end up dividing the cake into fractions. It is a hilarious story and students love it.	This story is about a frog who loves to read and the fireflies who love to listen. One night, the moon doesn't come out and the fireflies end up saving the day by aligning themselves into square numbers.	This is a magical book that allows different animals to spring out of the pages of the book. It talks about and shows different sizes of animals. Students are mesmerized when they read it.	This is a tongue-in-cheek tale of Descartes and how he came up with the idea for the Cartesian mapping system. It is a story well told.
Online resources	http://matthewmcelligott.com/lionsshare/projects.php http://mathgeekmama.com/the-lions-share-lessons-and-printables/	http://www.uen.org/Lessonplan/preview.cgi?LPid=18924	http://www.houghtonmifflinbooks.com/readers_guides/pdfs/JenkinsGuide.pdf	http://www.uen.org/Lessonplan/preview.cgi?LPid=11237
Activities	Have the students actually act out the story by cutting the cake (using a big piece of pink paper).	Have the students act out the story by modeling the square numbers with tiles.	Have the students measure the animal parts in both metric and customary measurements.	Have the students actually look at coordinate grids from malls, theme parks and subway schedules.
Questions to extend the learning	So, if the frog took half of ⅛, what part of the cake did he take? If the beetle took ½ of 1/16, what part of the cake did he get?	So, if the fireflies came down in a 4-by-4 array, how many fireflies came down? What if it wasn't bright enough and the fireflies had to come down in a 12-by-12 array? Can you model what that would have looked like on the grid paper and solve it?	How many centimeters shorter is the butterfly than the frog? What would a line plot of the length of all these animals look like?	Have the students locate different directions. For example, ask: "Which store is at B12?" Have the students make up questions using the maps.

Figure 8.2 Some More Great Math Picture Books by Topic

Place Value	**Fractions**	**Measurement and Data**	**Geometry**
Less than Zero—Stuart J. Murphy	*Fraction Fun*—David A Adler	*12 Snails to 1 Lizard*—Susan Hightower	*The Very Greedy Triangle*—Marilyn Burns
Betcha—Stuart J. Murphy	*Fraction Action*—Loreen Leedy	*Spaghetti and Meatballs*—Marilyn Burns	*When a Line Bends . . . A Shape Begins*—Rhonda Gowler Greene
A Million Fish . . . *More or Less*—Patricia C. McKissack	*Fractions in Disguise*—Edward Einhorn	*Measuring Penny*—Loreen Leedy	The Sir Cumference series: *Sir Cumference and the First Round Table;* *Sir Cumference and the Great Knight of Angleland*—Cindy Neuschwander
If You Made a Million—David M. Schwartz	*Gator Pie*—Louise Mathews	*Jim and the Beanstalk*—Raymond Briggs	*A Fly on the Ceiling*—Julie Glass
	Eating Fractions—Bruce Mcmillan	*Millions to Measure*—David M. Schwartz	*Cubes, Cones, Cylinders and Spheres*—Tana Hoban
	Piece=Part=Portion: Fractions=Decimals= Percents—Scott Gifford	*How Much Is a Million?*—David M. Schwartz	*Grandfather Tang's Story: A Tale Told with Tangrams*—Tompert Ann
	The Lion's Share—Matthew McElligott	*On Beyond a Million: An Amazing Math Journey*—David M. Schwartz	*Shape Up!*—David A. Adler
	The Hershey's Fractions Book—Jerry Pallotta	*Actual Size*—Steve Jenkins	*Mummy Math: An Adventure in Geometry*—Cindy Neuschwander
		Is a Blue Whale the Biggest Thing There Is?—Robert E. Wells	

Figure 8.3 Math Picture Books about Multiplication

Multiplication	**Division**
Amanda Bean's Amazing Dream—Cindy Neuschwander *The King's Chessboard*—David Birch *Each Orange Had Eight Slices: A Counting Book*—Paul Giganti Jr. *Sea Squares*—Joy N. Hulme *The Best of Times*—Gregory Tang *My Full Moon Is a Square*—Elinor J. Princzes *Math Curse*—Jon Scieszka *Anno's Magic Seeds*—Mitsumasa Anno *One Grain of Rice: A Mathematical Folktale*—Demi	*Divide and Ride*—Stuart J. Murphy *One Hundred Hungry Ants*—Elinor J. Pinczes *A Remainder of One*—Elinor J. Pinczes *The Doorbell Rang*—Pat Hutchins

Poems and Songs

There are many different types of songs and poems that teach mathematical concepts. *Songsforteaching.com* has great math songs about a variety of topics. *Canteach.ca* poems also has some really great poems about money. *Mathstory.com* also has excellent concept poems and songs.

Figure 8.4 Some Great Math Poems and Songs

Math Text	***One Inch Tall*—Shel Silverstein**	***The Metric Song*—Kathleen Carroll**	***Numerator Dog*—Mr. R**
Storyline	This is a hilarious poem by an all-time great . . . about being one inch tall.	This is a very clever poem about 2 families (the kilo's and the milli's). The students listen and learn about the different units of measure in the metric system.	This is a very funny poem about a dog that likes to get on top of things.
Online resources	http://www.marketplace.org/2009/04/27/life/poetry-project/poem-smart-shel-silverstein	http://www.songsforteaching.com/kathleencarroll/metricsong.htm	http://mathstory.com/Poems/mydognumerator.aspx#.Vt9aYWQrI6U

(*Continued*)

Figure 8.4 Continued

Math Text	*One Inch Tall*—**Shel Silverstein**	*The Metric Song*—**Kathleen Carroll**	*Numerator Dog*—**Mr. R**
	https://www.youtube.com/watch?v=y6AITlfWces https://sites.google.com/site/melissacookkindergarten/sample-lesson-plan-page		
Activities	Students can write their own versions of being one inch tall and then also extend that conversation to being 1 cm tall or 1 decimeter tall.	Students can illustrate this poem and then make a photo essay discussing and illustrating each of the units of measure.	Students then should look at numerators and define and illustrate what they mean.
Questions to extend the learning	What are 3 things that we can measure in cm? What are 3 things that we can measure in inches?	What are 3 things that we measure in millimeters? What are 3 things that we measure in kilos?	Write and illustrate 3 different fractions and explain what the numerators are in these fractions?

Believe It or Not Stories

Believe it or not stories are true, wild stories about intriguing animals (see example in Figure 8.5).

Figure 8.5 Flying Foxes and Megabats

Flying Foxes Megabats	
	
https://en.wikipedia.org/wiki/Pteropus; images from Dollar Photo	

Large flying foxes are the largest bats in the world. They live in colonies, sometimes with 10,000 to 20,000 in one colony. They fly up to 50 km (or 31 miles) to eat in one night. These fascinating creatures are called megabats and have a face like a fox and the body of a bat. They weigh between 0.65–1.1 kg or 1.4–2.4 lbs. They have a wingspan of up to 1.5 m or 4 ft. 11 in.	Questions: If a group of bat scientists found 5 colonies with around 20,000 bats in each, about how many bats would they have seen? If the flying fox traveled 31 miles each night for 7 nights, how far would he have traveled in a week? There were 3 flying foxes. The first fox weighed 0.79 kg. The second fox weighed 0.50 more than the first. The third fox weighed 0.10 less than the first bat. Make a bar diagram to illustrate the differences in the bats' weight. How much did they weigh altogether?

Word problem tic-tac-toe is another way to get students working with word problems. It is a game of tic-tac-toe with a twist. The students have to answer word problems to get an *X* or *O* in the squares (see Figure 8.6).

Figure 8.6 Word Problem Tic-Tac-Toe

The first American soccer league, the USSA, paid its players 35 cents for every goal scored. *If someone scored 7 goals, how much did they make?*	A professional soccer player runs around 6 miles in an average soccer game. *If a team practices 5 days a week for 4 weeks, about how far has a soccer player run?*	The first official set of soccer rules was written in 1863, although the game dates back over 2000 years ago to China. *How many years ago were those rules written?*
The World Cup is the biggest soccer tournament in the world. It happens every 4 years. *If there was one in 1998, when will the next one be?*	The rules state that soccer must be played on a rectangular field that is between 100 and 130 yards long and between 50 and 100 yards wide. *What is a possible area of a field?*	Kid's soccer balls cost \$9.99 each. *How much would 7 soccer balls cost?*
More than one billion fans watch World Cup soccer on TV. *Can you write that number?*	Soccer cleats cost \$64.78 each. *How much would 8 pairs of soccer cleats cost?*	The Women's United Soccer Association played its first game in 2001. *How many years has it been around?*

Fascinating Facts

Fascinating facts are another way to get students to love word problems. Pick some fun true facts and have the students make up problems based on those facts (see Figure 8.7).

Figure 8.7 Fascinating Facts: Elephants

 *Picture: Dollar Photo	• **Elephants are fascinating creatures. There are 2 types of elephants: African elephants and Asian elephants.** • **Elephants are the largest land animals in the world.** • **The largest elephant weighed 24,000 lbs! Wow!!** • **Adult elephants drink 210 liters of water a day.** • **Elephants can live to be over 70 years old.** • **African elephants can weigh up to 6,000 kg or 6.6 tons.** • **The elephant's trunk can be up to 2 meters long.** • **Elephants can spend up to 16 hours a day looking for and eating leaves, grass, twigs and roots.** • **Baby elephants weigh 230 lbs!**
Read the fascinating facts. Write 3 different word problems about elephants based on the facts.	**http://www.sciencekids.co.nz/sciencefacts/animals/elephant.html; http://www.animalfactguide.com/animal-facts/african-elephant/; http://www.happyelephantcontest.com/fun-facts/**
Problem 1:	**Problem 2:**
Problem 3:	**Bonus Problem**

http://www.sciencekids.co.nz/sciencefacts/animals/elephant.html; http://www.animalfactguide.com/animal-facts/african-elephant/; http://www.happyelephantcontest.com/fun-facts/

Real-Life Shared Experiences

Shared experiences are activities that students do to understand and play around with the concepts. For example, every year at one of the schools I worked at in the Bronx, we would have a math Olympics to reinforce measurement concepts. All of the upper elementary grades participated, and they would get class winners who would then represent them in the finalist competitions. We had wonderful games like *How far can you blow the cotton ball?* The students had to blow the cotton ball across the table through a straw in 4 puffs. We would then measure how many inches or centimeters they blew their cotton balls. We would have a meter jump—which entailed students jumping and then measuring to see who jumped the farthest. We would do things like milliliter races where the students had to fill a small glass bottle with an eyedropper. It was just a variety of games that were kid-friendly, academically-rich and extremely engaging. Since everybody did it, we could then relate to these units of measure and make up problems about them that everyone understood.

The teachers and the students would write word problems about the different events. For example:

1. What is the best unit of measure to weigh a small rock?
2. Could you use pounds to talk about how far you jumped?
3. If John jumped 89 cm his first jump, 99 cm his second jump and 1.5 m his third jump, then how far did he jump altogether? Was it more or less than 3 m, explain how you know?
4. If Maribel jumped a total of 2 yards, how many feet did she jump?

Key Points

- Math mentor texts are great shared experiences.
- Poems and songs can springboard into interesting math topics.
- Believe it or not stories help students to make sense of math in the real world.
- Shared experiences provide great opportunities to apply math.

Summary

Getting all students to *love* word problems means that they are working on problems that interest them. Those problems are not necessarily in the textbooks. That is not to say students shouldn't be able to solve the problems in the textbook. But, they have to learn the math and be able to understand it and be comfortable diving in and pulling it apart in new contexts. So starting from a place of familiarity boosts engagement and confidence. Confidence with competence makes for success.

Reflection Questions

1. Do you use math mentor texts to enhance and build knowledge around each big idea?
2. Do you use poems and songs to help teach the big ideas?
3. In what ways could you use *believe it or not* stories to grab your students' interest? How will you keep it once you have it?
4. What types of shared experiences do you have with your students that build on their math understandings?
5. What is your biggest takeaway from this chapter?

9

Math Literature Problem-Solving Circles and Other Collaborative Activities

Math Literature Circles

Collaborative problem solving is a great way to get students talking about word problems. There are many ways to do this. In this chapter, I want to talk about some of those ways, including math literature circles as a much-needed variation of group problem solving. A math literature circle is a small group of students gathered together to discuss a math problem in depth. The discussion is guided by the students' responses to a problem that they have already worked on. You may hear talk about how they visualized it, how they summarized it, their plan, the strategies they used, the models they used and how they double-checked it.

Math literature circles provide a way for students to engage in critical thinking and reflection as they read, discuss and respond to books. Working together on shared learning experiences are the center of this approach. Students listen to, question, watch and learn from each other. Their understanding of the mathematics deepens as they work together to discuss the problems. A structured discussion protocol scaffolds the learning experience.

Math circles are student-centered math discussions where each student has a distinct role. They meet at scheduled times during the week to work together. This ongoing activity occurs regularly, for example, every Friday. Before the meeting, students have done some work on their own so they may fully contribute to the discussion. This is a scaffolded conversation so that students are having a hands-on, minds-on fully engaged experience.

This student-centered approach focuses on students' responses to the math problems they encounter. In math literature circles, students are actively engaged in making sense of the problems that they chose to work with. They think about the problem, write about it, reflect on it and construct their own meaning before they meet with their group. When they meet with the group, each person comes with their thoughts about the problem and then discusses and co-constructs meaning with their peers. The students engage in Depth of Knowledge (DOK) (Webb, 1999) level 3 and 4 type activities, meaning that they are thinking strategically and reasoning, justifying and generalizing about the math. These discussions are guided by the students' thoughts, insights, observations and questions. The discussion is around the type of problem, the initial solving of the problem, the models used, the strategies used and the double-checking that takes place. The students approach the discussion through acting out different roles. They learn to facilitate their own discussions through scaffolded protocols.

The goal of math literature circles is to provide an enthusiastic, engaging, student-driven space for conversation that encourages deep thinking about math. The belief is that as students make sense of math together, the math begins to make more and more sense to the students.

Math literature circles begin with the idea that you take a really rich problem and give it to students the way you would a good book. Students read the problem and work through it on their own for a day or two and then they prepare to meet with their group. When they meet, they bring their work and come prepared to discuss their thinking so far on the problem.

Each person is given a specific role to work through the problem:

- summarizer/verbalizer/presenter
- equation maker
- planner/strategist
- modeler
- double-double-checker

Summarizer/Verbalizer

This person is in charge of reading the problem and explaining the meaning. Their role is to facilitate a discussion about the meaning of the problem, the type of problem and what is being asked. This person is also the lead presenter of the problem and the solution to the class.

Equation Maker

This person is in charge of reiterating and verifying the type of problem. They must decide if it is an addition, subtraction, multiplication or division problem. They have to decide on the type—what is missing—the beginning, the change or the start. They have to discuss whether it is one-step, two-step or multistep. They must then write a set-up equation. This is an extremely important role because it sets the path that everyone else will follow. The equation maker is also the person who writes the solution equation.

Planner/Strategist

This person is in charge of planning the problem out loud with the others. They do not do it in isolation but rather lead the discussion about what models and which strategies to use. This person should have a "strategist booklet" where they can reference various models and strategies to choose from. The planner/strategist has to be a good listener and facilitator of the discussion so that everyone's ideas are heard and considered in the discussion. They do make the final decision though.

Modeler

This person is in charge of doing the actual modeling. They can use concrete, pictorial or abstract models. For example, they might model with fraction strips and then draw a model of that or glue paper strips on to the problem. They are responsible for modeling one way and checking another. Sometimes, they work in collaboration with the equation maker because an equation is a model. The modeler needs to make sure that the equation matches the model.

Double-Double-Checker

The double-double-checker is responsible for checking the math and the answer. The double-checker is responsible for leading the discussion with the group around the accuracy of the calculations and the accuracy of the answer. This person actually needs to go back and reread the question to the group and then the answer to see if it all makes sense.

Class Presentation

After students have gone through this process, they must present their problem to the class. Sometimes this is done verbally and at other times this is done through a gallery walk where everybody goes around with Post-it® notes and comments on the different work they see. Of course, students should have a criteria list so they can give specific feedback.

Four Square Problem Solving

Another opportunity to do group problem solving is to have the students do a progressive problem (see Figure 9.1). In this type of problem, the teacher gives the students a word problem and four different colored pens.

For each rotation, the students are instructed to use a different color. The students spend the first seven minutes working on the problem on their own with the blue pen. They do this work in Square 1. Then they find a partner to continue unpacking the problem. They work with their partner on the problem in Square 2 using a green pen. After seven minutes, they move to a group of three or four students. In this group, they share their thinking with each other and they take notes in Square 3 about any new insights they have in orange. After seven minutes they come back to the whole group. The whole group discusses what they did and how they did it. Students use Square 4 to note any new ways to think about the problem in purple.

This progressive problem solving can be very productive and get students to think about problems in multiple ways. I first learned of it from my friend Kimberly Mayfield (personal communication, 2003). She had done this in a college course as part of a progressive quiz. This is actually a great way to get students to practice problem solving. It allows them to not only explain their thinking but also listen to and process the thinking of others in relation to their thinking. The recording sheet allows the student to keep track of the growing conversation and information. It also provides an insight for the teacher into what was happening during the discussions. The *Teaching Channel* has a great video and template for doing something similar to this. They call it Choose 3 Ways. In this model, the student does it on their own, and they try to do it three ways and then they get with a group. (The video can be seen here: https://www.teachingchannel.org/videos/problem-solving-math.) The worksheet is called *Choose 3 Ways*. You can Google the worksheet.

Figure 9.1 Four Square

1. Individual: Solve the problem by yourself. What strategies and models will you use?	2. Partner: Work with a partner to show a different way.
3. Group: Talk with the group about how you have thought about the problem so far. What new ideas do you have after this discussion?	4. Whole Class: What did you hear during the whole class discussion that you could add to your thinking?

Tell Me a Story

This storytelling structure allows students to make up word problems (see Figure 9.2). In this activity students pick a story, pick some numbers and then write the problem. Other times the students are given the context and the problem. The students each have roles. Someone starts the story, someone explains it, another person models it and another person double-double-checks the answer.

Figure 9.2 Tell Me a Story

1. The answer is 4 marbles. It is a division problem. What is the story? 	2. Write the story.
3. Write a set-up equation: __________ ______________________________	4. Make a plan.
5. Model your thinking.	6. Write a solution equation.
7. Check your work.	8. Explain what you did.

Word Problem Rework

Christine Mulgrave-King writes about doing the word problem rework (see Figures 9.3 and 9.4). In this activity, students have a starter problem and then they have to rework that problem. To rework the problem, they can

- Change the numbers
- Change the operations

- Change the story (use same numbers and same operations)
- Change the language
- Change the units

This can be so powerful to have students do. They should get a chance to practice doing this during whole class problem-solving routines so that they understand how to do it. Also, practice this in small guided math groups by doing one problem with the group like this but also giving each student their own problem to rework and then to explain to the group how they did it. Finally, have the students also do this in the word problem workstation. They can do it alone, with a partner or in a group.

Figure 9.3

> Christine Mulgrave-King has some brilliant ideas about doing word problem work. She has written several books, but two in particular that relate to problem solving are *Test Savvy Math: Fostering Thinking and Reasoning into the Test-Prep Process* (which is a book that focuses on integrating test prep throughout the year with meaningful routines and activities rather than a frenzied month before test prep craze). Another book is *12 Strategies for Understanding Word Problems.*

Figure 9.4 Word Problem Rework

Original problem: The kids ate ⅓ of Grandma's cake in the afternoon. Then, later that evening they ate ⅓ more. How much was left?
Rework 1: The kids ate 2⁄4 of Grandma's cake in the afternoon. Then, later that evening they ate ¼ more. How much was left? (changed the numbers)
Rework 2: Grandma baked some cupcakes. She decorated ⅓ of them with chocolate chips. She decorated ⅓ of them with strawberries. How much of the cupcakes did she leave plain? (changed the story)

Key Points

- *Math Literature Circles* promote deep thinking and talking.
- *Four Square* helps students to learn from each other.
- *"Tell me a story"* gets students to pose word problems.
- *Word Problem Rework* (by King, 2013a) helps students to think about the problem in many ways.

Summary

Collaborative problem solving should be a frequent activity in the classroom. Students should be encouraged to think together and discuss each other's approaches. They should reason about what is being said and if it makes sense and if there are other ways that might be more efficient. There are different ways to scaffold this, including math literature circle work, four square work, collaborative storytelling and word problem rework (King, 2013a). When students are working together, it is important to scaffold their work, giving them roles and templates so that the efforts are organized and productive. One of the biggest parts of making this work is reviewing the guidelines for accountable talk so that students are helpful rather than hurtful when discussing the ideas of others.

Reflection Questions

1. In what ways do you get your students to work together on word problems?
2. Do you currently do anything like a math literature circle, four square, collaborative story writing or word problem rework?
3. What do you see as some of the major benefits and possible challenges of having students collaborate on word problems?
4. What is your biggest takeaway from this unit?

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10

Schoolwide Efforts

The Great 100 Word Problem Challenge

Every year, thousands of schools do some version of the 100 Book Challenge®. This great program has students read 100 books throughout the year, and they get all sorts of prizes and rewards along the way to cheer them on. Students get excited about reading and everybody gets involved. The Great 100 Word Problem Challenge is created in the same spirit. It is a fun way to scaffold student engagement and success with word problems. It is exciting, challenging and rewarding.

The Great 100 Word Problem Challenge is an individual student challenge to solve 100 word problems by the end of the year. It is divided into 10 levels, each level consisting of 10 problems (see Figures 10.1 through 10.11). Students can win prizes at each level. The levels grow progressively more difficult, with the first two levels being a general review of the word problems from prior grades. The third level focuses on number lines. The fourth level focuses on students' choices as they navigate through

Figure 10.1 The 100 Word Problem Challenge

Level 1 Solve problem using Template A. *Focus is on mathematical sketches*	Level 2 Solve problem using Template B. *Focus is on tape diagrams*	Level 3 Solve problem with Template C. *Focus is on number lines*	Level 4 Graduated level of difficulty A. choices given B. students choose numbers	Level 5 Two-Step Word Problems
Level 6 Multistep word problems	Level 7 Write and solve word problems.	Level 8 Sort word Problems.	Level 9 Solve two-argument problems.	Level 10 Find and fix the error.

grade-level problem types. The fifth level emphasizes two-step problems and the sixth level emphasizes multistep problems. At the seventh level, students focus on writing their own word problems. At the eighth level, students work on sorting all the different problem types as well as if the problem is one-step, two-step or multistep. The last two levels focus on looking at the thinking of others and deciding whether or not the arguments make sense, and if not how to fix the errors.

Figure 10.2 Third Grade Example: Level 1—Template A

Read the problem below and make a picture in your head. *Mary had $25. She got some money for her birthday. Now she has $75. How much money did she get?*	
What type of problem is this? Addition Subtraction Part-Part-Whole Compare	What are we looking for? Write an equation with a symbol for the part that we are looking for.
Model the problem with a sketch.	
Write the equation with the missing number in it. Answer: ________________________units	

Figure 10.3 Third Grade Example: Level 2—Template B

Read the problem below and make a picture in your head. *Mary had some money. She got $50 for her birthday. Now she has $175. How much money did she have in the beginning?*	
What type of problem is this? Addition Subtraction Part-Part-Whole Compare	What are we looking for? Write an equation with a symbol for the part that we are looking for.
Model the problem with a bar diagram.	
Write the equation with the missing number in it. Answer: ________________________units	

Figure 10.4 Fourth Grade Example: Level 3

Read the problem below and make a picture in your head. *Sue went skating at 4:15. She was gone for 3½ hours. What time did she come home?*	
What type of problem is this? Addition Subtraction Multiplication Division	What are we looking for?
Model the problem with a number line diagram.	
Answer: ______________________units	

Figure 10.5 Fourth Grade Example: Level 4

Read the problem below and make a picture in your head. Choose numbers and fill in the blanks. *The bakery sold _______ boxes of cookies. Each box had ___________ cookies in it. How many cookies did the bakery sell altogether?*	
What type of problem is this? Multiplication Division	What are we looking for? Write an equation with a symbol for the unknown.
Model the problem.	
Write the solution equation with all the numbers filled in. Answer: ______________________units	

Figure 10.6 Third Grade Example: Level 5

Read the problem below and make a picture in your head. *Mary had $199. She got some more money. Now she has $325. How much money did she get? If she needs $500 for her new bike, how much more money does she need to save?*	
What type of problem is this? One-Step Two-Step	What are we looking for? Write an equation(s) with a symbol for the part that we are looking for.
Model the problem.	
Write the equation(s) with the missing numbers in it Answer: ______________________units	

Figure 10.7 Fourth Grade Example: Level 6

Read the problem below and make a picture in your head. Mike had 4 boxes of marbles. Each box had 15 marbles. For his birthday, his brother gave him 7 more marbles. His sister gave him twice as many as his brother. How many marbles does Mike have now?	
What are we looking for first? What are we looking for second? What are we looking for next?	
Model the problem.	
Answer: ________________________units	

Figure 10.8 Fifth Grade Example: Level 7

Write a word problem for the equation below. $^{30}/_{5}$	
Write the problem.	
Model the problem.	
Answer: ________________________units	

Figure 10.9 Fifth Grade Example: Level 8

Read the word problems below and decide what type of problem they are. Circle the type.				
Bob had 50 marbles. He got some more. Now he has 78. How many did he get?	There were 10 rows of cookies in the bakery. Each row had 7 cookies in it. How many cookies were there altogether?	Maria had 20 marbles. Her cousin had 4 times as many as Maria. How many did her cousin have?	There were 456 kids in the cafeteria: 289 were girls and the rest were boys. Some more boys came. Now there are 200 boys in the cafeteria. How many boys came?	Mark had 15 marbles. He had 3 times as many as his friend. How many does his friend have?
Addition, Subtraction	Multiplication or Division	Additive Comparison Multiplicative Comparison	One-step Two-step	Two-step Multistep

Figure 10.10 Third Grade Example: Level 9

John said he ate ½ of a candy bar. Mike said he ate ¼ of a candy bar. Mike said he ate more than John. John said that isn't possible because ½ is greater than ¼. Who is correct and why?
Explain your thinking with numbers, words and a model.

Figure 10.11 Fourth Grade Example: Level 10

Clint solved $\frac{4}{5} + \frac{4}{5} = \frac{8}{10}$. Can you find his error, explain it and fix it?		
Find it.	Explain it.	Fix it.

Key Points

- There are 10 levels to the Great Word Problem Challenge.
- There are various types of problems including one-, two- and multistep problems.
- There are various models including strip diagrams, drawings and open number lines.
- Students should be reasoning about the thinking of others.

Summary

Students need to be motivated. We have got to find engaging ways to get them to *love* word problems. Having them participate in some sort of activity where they are competing with themselves to learn more can be highly engaging, extremely effective and exciting. The payoffs are incredible. Try the Great Word Problem Challenge in your school!

Reflection Questions

1. Could you see yourself doing this with your students?
2. What do you think the benefits are of doing something like this?
3. How might your whole school gain from participating in something like this?

11

Assessment

We must bring a different type of intentionality and focus to assessing students' word problem skills and strategies, if we want them to get better at it.

Problem solving is one of the top two difficulties that students have in schools. Fluency is the other. If we could conquer those two mountains, we would be able to move student achievement forward by great leaps. If we are really concerned about these things, we must track them. If we track them, then we have a better handle on them. We know where to start and where to go.

At the beginning of the year, in every grade you should give a word problem test. This test should assess students' problem-solving skills from the grades before. In the middle of the year, the test should be repeated with the on-grade level problems. At the end of the year, the test should be given again, this time testing all of the problems that were supposed to be mastered in that grade. This information is vital if you want to improve problem solving in your class, school and district. With this information, it is possible to look at data by individual, within the class, across the grade, across the school and across the district. Then it is important to think about where are the problems? Why are they there? How often do the students get to practice these types of problems? Let's take a deeper dive into this.

Here is an example of some of a 3rd grade ***Beginning of the Year Benchmark*** (see Figures 11.1, 11.2 and 11.3). Since it is a beginning of the year test, these are second-grade problems. The point is to see if there are any gaps in second-grade problem solving before we dive head first into third-grade problem solving. Notice that the students are required to represent their thinking with drawings and diagrams.

Figure 11.1

Draw and Solve.

1. There were 25 bugs on the rock. Then, some more crawled on to the rock. Now there are 57 bugs on the rock. How many bugs crawled on to the rock?

Answer Box

Figure 11.2

Draw and Solve.

2. There were 307 boys and 287 girls in the school cafeteria. How many fewer girls were there than boys? How many children were there altogether?

   A. Show your thinking with a tape diagram.

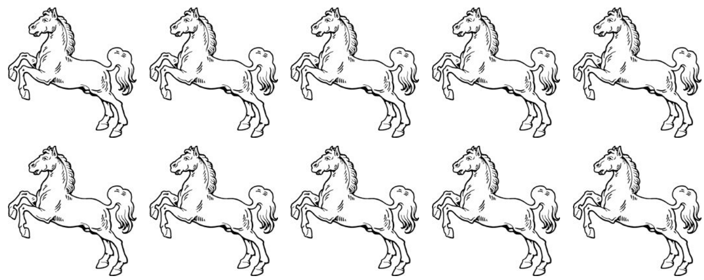
   B. Explain your answer.

Answer Box

Here is an example of some parts of a Fourth Grade Word Problem Midyear Benchmark (see Figures 11.4, 11.5 and 11.6). Notice that the students are asked to model their thinking with a drawing, number line or tape diagram. It is important that students show their knowledge of the designated models of their math state standards. This test also requires that students write a set-up equation with a symbol for the missing part. All state standards require that students learn how to do this starting in first or second grade. However, most classroom assessments tend to look for a solution equation rather than require students to write set-up equations.

Figure 11.3

3. Use the pictures to solve. There were 10 horses in the grass. They were lined up in groups of 2. How many groups were there?



Answer Box

Figure 11.4

1. Katie ate ¼ of a candy bar. Then she ate some more. Now she has ²⁄₄ left. How much did she eat?
   a. Model your thinking with a drawing, number line, or tape diagram.
   b. Write an equation using a symbol for the missing number.
   ____
   c. Answer ______________________

The question in Figure 11.5 illustrates that there must be open questions as well (meaning that students can use whatever models they choose). There should be a good balance on a word problem assessment between students choosing their models and showing that they know how to use the designated models.

Figure 11.5

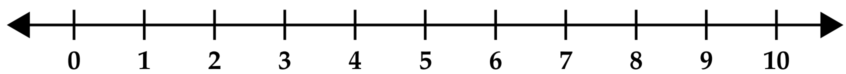
1. Mario had 10 marbles. His brother had 2 times as many as he did? How many did they have altogether?
   a. Model your thinking:
   b. Answer _________________

The next question in Figure 11.6 gives another example of using a designated model (the number line).

Figure 11.6

1. Ken ran 2/10 of a mile in the morning. He ran 5/10 of a mile in the afternoon. He ran 1/10 of a mile in the evening. How far did he run altogether? Did he run a full mile?

   A. Model your thinking with the number line.



   B. Write an equation with a symbol for the missing number.

   ____

   C. Answer: ____________________

Here is an example from a Fifth Grade Midyear Word Problem Benchmark (see Figure 11.7). Notice in this test the students are required to pose a problem. They need to contextualize the numbers.

Figure 11.7

1. The answer is $\frac{5}{6}$ of a pie. Write a word problem about this.

   a. Problem:
   b. Model your thinking:
   c. Answer: __________________

Many Parts

There are many different aspects of answering word problems that should be considered when writing and giving word problem assessments. Teachers should be concerned with more than just getting a correct answer. The answer is necessary but not sufficient. We have to know if students can model their thinking, communicate their understanding, show what they know in more than one way, discuss their strategies, use the correct math vocabulary, persevere and answer all the parts of the problem, and reason about the numbers.

Sometimes what happens is students can answer the question but can't model their thinking or discuss their strategies. This means that the answer is incomplete. Teachers will contact me and say, "Nicki, the kids could answer the question but they couldn't do the tape diagram." To which I reply, "Well then they could only answer part of the question and so they get partial credit because being able to model one's thinking is part of mathematical proficiency." So, we have to get better at learning and being completely comfortable with the models and strategies that the new state standards across the U.S. are asking students to know and be able to do.

Analyzing the Data

So, when the data is collected, teachers should notice what is happening. The data has to be analyzed and interpreted. Maybe as in the case just mentioned, most of the students can't make a tape diagram yet. This is an issue. If it is a whole class issue, then it needs to be addressed in the whole class problem-solving routine. If it is an issue with some of the students, then it should be addressed in the small guided math groups.

Here is an example of a fourth-grade data analysis sheet (see Figure 11.8). It is an Excel sheet so that the calculations can be easily scored and the percentages given and groups formed accordingly. With this type of form, teachers are able to see who is missing which questions. The actual error analysis allows teachers to then pull small-guided math groups and address the specific error patterns of the students. The groups will be sorted into experts, practitioners, apprentices and novices. The experts are the students that are working above grade level. The practitioners are the students who are on grade level. The apprentices are approaching grade level and the novices are working below grade level. The implications of this information is that guided math groups are pulled accordingly and math workstations have differentiated activities. So everyone is not solving the same problem at the same time, always. Sometimes they are solving the same problem. This occurs when students are working in groups or with partners. Usually, they are working with partners at a similar level. Sometimes problem-solving groups are heterogeneous and sometimes homogeneous.

The Problem with Word Problems

The research points out specific problems that students have with story problems. There is the comprehension phase and the solving phase. During the comprehension phase, "problem solvers process the text of the story problem and create corresponding internal representations of the quantitative and situation based relationships expressed in the text" (Nathan et al., cited in Koedinger). During the solution phase, students try to find the solution. Several researchers have found that errors in the comprehension phase account for problem-solving difficulties (Lewis &

Figure 11.8

Class:________ Teacher:________ Date:________

Domain: Operations and Algebraic Thinking, Number & Operations—Fractions & Measurement and Data														
Fourth Grade End of the Year Benchmark Word Problems Data Sheet														
Standard: 4.OA.2: Multiply or divide to solve word problems involving multiplicative comparison. **4.OA.3:** Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding**. 4.NF.3:** Understand a fraction a/b with a > 1 as a sum of fractions 1/b. **4.NF.4:** Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. **4.MD.2:** Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale. **4.MD.3:** Apply the area and perimeter formulas for rectangles in real world and mathematical problems. **4.MD.7:** Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems.														
Questions	**Q1**	**Q2**	**Q3**	**Q4**	**Q5**	**Q6**	**Q7**	**Q8**	**Q9**	**Q10**	**Q11**	**Q12**	**Total**	**% Correct**
Points	1	1	1	4	2	1	1	3	2	2	1	1	20	100.00%
Standards	4.OA.2	4.OA.2	4.OA.2	4.OA.3	4.OA.3	4.NF.3	4.NF.3	4.NF.4	4.MD.2	4.MD.2	4.MD.3	4.MD.7		
Students														
Lucy C.	1	1	1	1	1	1		2	1	1	1	1	12	60.00%
Carol M.	1	1	1	4	2	1	1	2	1	2	1	1	18	90.00%
Luke B.	1	1	1	3	1	1	0	1	1	1	1	1	13	65.00%
Mike M.	1	1	1	1	1	1	1	1	1	1	1	1	12	60.00%
Juan R.	1	1	1	3	2	1	1	2	2	2	1	1	18	90.00%
Kelly E.	1	1	1	2	2	1	1	2	2	2	1	1	17	85.00%

National Governors Association Center for Best Practices & Council of Chief State School Officers. (2010). Common Core State Standards for Mathematics. Washington, DC: Authors.

Mayer, 1987; Cummins et al., 1988). Specifically, many of the errors in comprehension have to do with language.

In 1977, Australian educator Anna Newman discussed five steps that students need to work through in order to solve a word problem successfully:

1. reading the problem/READING
2. comprehending what was read/COMPREHENSION
3. transforming the words into a numerical representation/ TRANSFORMATION
4. doing the calculations/PROCESS SKILLS
5. writing and explaining the answer/ENCODING

Her research showed that over 50 percent of errors that children make occur in the first three steps—before they even begin to do the calculations! She suggested a five-step protocol for word problem solving error analysis. She would ask the following questions. The five questions link to the five processes (noted alongside them). Wherever the student has a breakdown, this is where the teaching point begins. Now, if asked to redo the problem and the student gets it right and can self-correct, Newman labels this as a careless error. All other errors are teaching points.

We can learn so much from students' error patterns! We need to look closely at what they are doing and what they are not doing. White (2005) states, "Mistakes can become entrenched, so error analysis is the first step towards doing something relevant which will remove the cause of the mistake." The Newman error analysis/interview protocol can help us figure out what is going on with students with word problems. White (2005) gives a great synopsis of the protocol. Here is a summary of that synopsis.

The Five Newman Questions/Requests

1. **Please read the question to me**: (This identifies **Reading Errors.**) Be sure to ask the student to tell you if they don't know a word. (Put an *R* if there are errors here.) **A reading error is when the student's reading prevents him or her from understanding the problem.** If they cannot read key words or understand key symbols in the problem so much so that it prevents them from understanding the problem, then this is classified as a reading error.
2. **Tell me, what the question is asking you to do.** (This identifies **Comprehension Errors.**) (Record a *C* if the student has problems.) **A comprehension error is when the student can read the whole problem but doesn't understand what to do.** They did not get the big picture of what was going on in the problem and therefore could not proceed with the problem.

3. **Which method do you use to get your answer?** (This checks for **Transformation Errors.**) (Record a *T* if there are problems here.) **A transformation error is when the student can read the problem, and comprehends what to do, but doesn't know how to do it.** The student doesn't know what operation(s) to use. They get stuck. For example, they don't know if it is a multiplication or a division problem.
4. **Show me how you get your answer, and "talk aloud" as you do it so that I can understand how you are thinking**. (This checks for **Process Errors.)** (Put a *P* if there are errors here.) **A process error is when the student can read the problem, comprehend it, knows what to do but can't do it.** It is when they don't know how to do it. For example, they know they need to multiply but they don't know how.
5. **Now, write down your actual answer.** (This checks for **Encoding Errors—**defined as **an inability to express the answer in an acceptable form**.) Ask the student to tell you the answer and to explain the answer. (Record an *E* if there are errors here.) **An Encoding Error is when the student finds the answer but can't write it out as the actual solution to the problem.** They don't know how to express it as the answer.

Remember that if the student self-corrects it could be classified as a **Careless Error** and coded with an *X*. Newman (1977, 1983) also said that students make errors due to a lack of motivation. Researchers have also found that 70 percent of word problem errors were at the comprehension and transformation levels (Marinas & Clements; Singhatat; Clements & Ellerton; cited in White, 2005).

Willis and Fuson (1988) provide another lens through which to look at student word problem errors. They give us four things to think about.

1. Can the students represent the problem? I would also ask, "What do they use?" Do they have a large repertoire of strategies for solving problems ranging from concrete materials to pictorial representations through abstract representations such as equations? Do they know how to use pictures, bar diagrams and the open number lines?
2. Do students understand the specific relations among the three problem quantities? In other words, can they put the numbers in the right place? Do they know what they know and what they don't know? Are they certain about what they are looking for? I find this to be especially interesting when I ask students, what does that number mean? What does it represent? What are you

trying to find out? Many students have trouble putting into words what the actual numbers mean in the problem.

3. Are they able to choose a correct solution strategy? Do they know which operation to use? Are they flexible with numbers? Can they see more than one way to do it? It is good to give them some multiple choice word problems where they have to pick the correct equation because this gets at the question above and this one. Do they know what the numbers mean and can they do the actual calculations? Also, it is good to have students solve one way and check another to see if they have that flexibility in working with the numbers.
4. Finally, can the students carry out the solution strategy correctly? Do they know how to do the actual calculations?

Student Reflection

Student self-reflection about their own problem-solving work is important.

At the beginning of the year, students should take a survey to see how they think about themselves as problem solvers (see Figures 11.9 and 11.10).

Figure 11.9 Word Problem Survey

1. What do you think of when you hear about word problems?

   Use numbers, words and pictures to describe your thinking.

2. Circle how well you think you can solve word problems.

1	2	3	4	5
great	good	okay	not good yet	still need lots of help

3. Do you like word problems?

   A. a little
   B. a lot
   C. not at all

   Explain your answer:

Figure 11.10 Word Problem Survey

Here are all the word problem types that you are expected to know this year. Which ones do you know how to solve? Which ones are you ok solving? Which ones are you still learning how to solve? Put a G for doing good, an ok for those that you are OK with and NY for those you haven't learned yet.

Multi-Step Problems with adding, subtracting, multiplying and dividing	Multiplication Multi-Step	Multiplication Array Groups
Two-Step Problems	Multiplication Equal Groups	Division Array Groups
Elapsed Time Problems	Multiplication (Comparison) Difference	Division Equal Groups
Customary Measurement Problems	Multiplication (Comparison) Smaller Part Unknown	Area and Perimeter Problems
Metric Measurement Problems	Multiplication (Comparison) Bigger Part Unknown	

Zdonek wrote about a way to get students to self-reflect on their quizzes (cited by Dyer, 2014). She had them fill in a form for each problem that they got wrong. First, the students had to rewrite the problem. This step ensured that they actually had to look at the problem and think about it. Next, the students had to try to solve it again (this time correctly). I would do this as a guided activity, talking through the various steps. Finally, they had to explain what they did wrong the first time, so they could reflect on the error patterns and learn from it.

Self-reflection allows students to think about where they are, where they want to be and how they are going to get there. Self-reflection should be scaffolded so that students can look at actual evidence about where they currently are and allow them to set goals of where they need to be. The goals should be SMART ones—specific, measurable, achievable, relevant and time-bound (Doran, 1981). For example, when reflecting on a quiz, students need to get specific about what they know and don't know (see Figure 11.11). They need to set a goal that can be measured around working on specific types of word problems that they missed. The time frame should be realistic (achievable) and the content (the actual word problems) relevant to the grade-level standards.

Figure 11.11 Word Problem Test Reflection

Name:

1. How did you do on the word problem test?
2. Which problems were easy?
3. Which problems were difficult? Why were they difficult?
4. What do you need to get better at?
5. What is your plan to get better?

Key Points

- Curriculum-based assessments are important.
- Do at least beginning of the year, midyear and end of the year assessments.
- Word problems should assess not only the content but also the processes/practices.
- There should be a specific word problem data collection plan.
- There should be analysis and interpretation of the error patterns.
- Anna Newman developed a five-point word problem running record.
- Willis and Fuson also gave us four points to consider when assessing word problems.
- Students should reflect on their word problem work.

Summary

Assessment is the key to learning. Our instruction must be driven by good assessments. Curriculum-based assessments help us to monitor where our students are throughout the year. Assessments must look at not only content but also processes. Teachers must analyze and interpret the specific errors that students are making and then design interventions that address those errors. Students must engage in ongoing and deep reflection about their problem-solving strengths and weaknesses. Word problems are notoriously difficult, but through a scaffolded program of intentional assessment, analysis, interpretation and reflection we can get all students to do well.

Reflection Questions

1. Do you perform a specific word problem assessment at least three times a year?
2. Do you collect the data and perform an analysis and interpretation of the specific error patterns?
3. Do you ever conduct word problem running records on your students?
4. Do you have your students reflect on their problem-solving skills and set goals to get better?

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12

Action Plan

Problem solving is an integral part of the mathematics learning. In everyday life and in the workplace, being able to solve problems can lead to great advantages.

(NCTM, 2000)

If you want your students to actually learn how to problem solve, you must have a plan that you implement. That plan consists of several parts, including assessments, a daily routine, guided math lessons, workstations, homework and possibly a schoolwide activity. Next are different templates to help you think about where you are currently and where you want to be (see Figures 12.1, 12.2, 12.3 and 12.4).

Figure 12.1 Reflection Grid About Problem Solving

Whole Class Routines	**Class Anchor Charts** ***Good; Okay; Needs Work***	**Class Toolkits** ***Good; Okay; Needs Work***	**Problem of the Day** ***Good; Okay; Needs Work***
Small Group Instruction	Do you have the data to support the work you are doing in small groups? *Good; Okay; Needs Work*	Do you do have a variety of activities in your word problem sessions? *Good; Okay; Needs Work*	How do you assess the work you are doing in small groups? What is the immediate feedback that you collect to see if you are accomplishing what you set out to teach and that the students are actually learning what you mean for them to learn? *Good; Okay; Needs Work*

Math Workstations Do your math workstations generally have a variety of activities including word problem sorts, matching/ concentration activities, posing and solving problems?	Individual Work/How do you provide for self-assessment? *Good; Okay; Needs Work*	Partner Work/ How do you scaffold peer editing? *Good; Okay; Needs Work*	Group Work *Good; Okay; Needs Work*
Assessment	Do you have individual data on Problem Type Proficiency/ Levels of students being able to solve the problems? *Good; Okay; Needs Work*	How do you collect ongoing evidence of student achievement throughout the year on problem solving? *Good; Okay; Needs Work*	Are you doing or thinking about doing word problem running records—at least on the students who are exhibiting extreme difficulty? *Good; Okay; Needs Work*
Your Own Knowledge	CGI Problem Types *Good; Okay; Needs Work*	Various Word Problem Activities *Good; Okay; Needs Work*	Knowledge about getting students to pose/write word problems *Good; Okay; Needs Work*

Figure 12.2 Word Problem Action Planning Mat

Objectives (List of objectives)	**Tasks (What you need to do to achieve your objectives)**	**Success Criteria (How you can identify your success)**	**Time Frame (When you need to achieve the tasks)**

Figure 12.3 Planning Template

Goal:			
Strategy:			
What steps must be taken to accomplish this strategy? 1 2 3	What is the timeline?	What resources do you need to do this?	

Figure 12.4 Word Problem Action Planning Mat

Purpose (Mission/Vision)	Current Reality/ What is the state of teaching and learning word problems in your class and school right now?	What are your plans to address these issues?	What do you want to happen? How do you want the reality to be different?
			Where do you want to be in the next year? What does it look like? What does it feel like? How does it sound?

FAQs

What should I do to help my kiddos get better at solving word problems?

Solve word problems. Daily, weekly, monthly. I'm serious. There is no magic bullet to teaching students how to solve word problems. It is consistency—which means over time, throughout the year. The emphasis has to be on the practice and process of solving problems rather than on getting the answer. The answer is necessary but not sufficient. We live in a world where students are expected to think about, solve and communicate with each other about different problems.

Is it a daily routine?
It has to be daily. That is a non-negotiable. But daily doesn't mean it takes up the whole day. Don't get sucked into the word problem for the entire math period. Go in, work with it a bit and come out. Rome wasn't built in a day and neither will problem-solving skills be. But, I promise if you work at it daily, students will learn how to do it well and become competent and confident problem solvers.

What does the problem-solving workstation look like?
The word problem workstation should have different activities where students have to think, solve and explain their work. There should be word problems that they have to solve and word problems that they have to write. They should have to check their peers' work. They should play different games where they have to decide what type of word problem it is or what is the most efficient way to answer it. The workstation should have paper and pencil as well as virtual opportunities for students to hone their problem-solving skills.

Do *I* really have to know the problem types?
Yes, Matilda, you do have to know the problem types. You have to know the problem types because you are supposed to be teaching them, and if you don't know them, then it is going to be a lot harder to teach them. The reason you must learn them is that they actually help you to teach them better and the students to learn them better. It takes the mystery out of problem solving.

We just teach the problems from the book, do the *kids* really have to know the problem types?
Yes, the students should learn to think about the problems conceptually. The research shows that when students know the problem types, they do better because they are focusing and breaking down the problem. They should also be exposed to problems from their everyday lives. They should be writing problems. So workbook problems are just that. They aren't all bad, but they can't carry the whole problem-solving curriculum.

Where do I start?
Start anywhere you feel comfortable. Start small. Start with a whole class daily routine. Start by trying to figure out who knows what in your class. Start by pulling small groups and working with them on the problems they are having trouble with. Yes, do go back and teach one-step problems, if that's what your students need (even if they are in the fifth grade). Hopefully, it'll be quicker and you can get to those multistep problems with the students understanding what they are doing. Sometimes, we have to slow down to speed up!

How do I monitor progress throughout the year?
Definitely give at least a beginning of the year, middle of the year and end of the year word problem exam. This will at least give you some useful information to navigate your teaching throughout the year.

What about time? There just is not time! How do I do this during *my* math block?
Time is always a question. Cut some ELA time (they usually have double the math block). Okay, I digress. You have to make the time to teach problem solving in a planned manner. That means that you must track what you are doing, so that you actually get to all the problem types for your grade level during the year. You should make a big problem-solving map/plan for your class, the grade and the school. You need to be very intentional about the homework as well. Always send home some word problems during the week for students to work on. Also, always have a word problem workstation up and running throughout the year.

Reference

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics.* Reston VA: NCTM. Retrieved August 4, 2015 from http://mrflip.com/teach/resources/NCTM/chapter3/numb.htm.